MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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W. H. MAW, Esq., PRESIDENT, in the Chair.

Frederick Joseph William Crowe, Marsden, Chichester ;
Thomas Edward Heath, 53 Park Place, Cardiff ;
Samuel Thomas Johnson, Melrose, Bridport, Dorset ;
George Tyrrell McCaw, Geodetic Survey, North Eastern Rhodesia, South Africa ; and
Percy Alfred Talbot, B.A., F.R.G.S., F.A.I., Sapele, Southern Nigeria, West Africa,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Edmund Dickson, F.G.S., 2 Starkie Street, Preston, Lancashire (proposed by A. L. Cortie) ;
Jacob Karl Ernst Halm, Ph.D., Royal Observatory, Edinburgh (proposed by F. W. Dyson) ;
John William Hicks, Computer to the Ordnance Committee, Ordnance Committee Office, Woolwich (proposed by Sir W. H. M. Christie) ;
William Malin Hunt, engineer, 48–50 London Road, Nottingham (proposed by Samuel Fellows) ; and
Lieut. Edwin Baikie Simpson-Baikie, R.N.R., 142A, The Bluff, Yokohama, Japan (proposed by D. P. Richards).

A hundred and one presents were announced as having been received since the last ordinary meeting, including, amongst others :—

Miss A. M. Clerke, Modern Cosmogonies, presented by the author ; W. T. Lynn, Remarkable Comets and Remarkable Eclipses (new editions), presented by the author ; Paris Observatory Memoires, vol. xxiv. (Gaillot, Tables de Saturne), presented by the Observatory ; Philadelphia Observatory Publications, vol. ii., pts. 2, 3 (Measures of Double Stars &c.), presented by the Observatory ; Tōkyō Observatory Annals, vol. iii., pts. 3, 4, presented by the Observatory.

Royal Observatory, Greenwich, eight original negatives of the Sun taken 1905 October–November, and thirty-six charts of the Astrographic Chart of the Heavens, presented by the Observatory ; photographs of the Solar Eclipse of 1905 August 30, presented by Mr. E. W. Barlow ; two photographs of the Flash Spectrum, 1905 August 30, presented by Dr. S. A. Mitchell.

On the Correction to Hansen's "Tables de la Lune" as deduced by Mr. Cowell. By E. Nevill.

As Mr. Cowell has brought to a conclusion the series of papers he has been contributing for the last two years to the *Monthly Notices* on the errors of the lunar tables, as shown by the Greenwich observations since 1750, it seems fitting that one who has been working for thirty years on the subject should offer such criticism as seems necessary on the results that have been obtained in one of the most important contributions that have been made for years to the better understanding of the motion of the Moon.

(1) The methods of investigation adopted by Mr. Cowell are perfectly sound, and the apparent values of the corrections he has deduced from the observations and embodied in Table X. of his paper in the *Monthly Notices* for 1904 December appear to have been derived with accuracy in a legitimate manner. Some small errors may have been made and some mistakes overlooked ; but in so long and complicated an investigation such things are unavoidable, as I have learnt by much experience. My own investigation of the same observation, completed years ago, have afforded many facilities for testing the accuracy of Mr. Cowell's work, and have shown its freedom from important error ; and I have no hesitation in accepting the values given in Mr. Cowell's Table X. for the apparent coefficients of the *sine* and *cosine* of the various arguments considered, as the actual values corresponding to the adopted date.

The criticisms I have to offer deal with the proper interpretation to be put on the results which have been deduced from the

analysis of the observations, and to the consequent conclusion which should be drawn. In cases Mr. Cowell has failed to do justice to his own results, and has drawn his conclusions in a manner open to improvement and to extension, whilst in others he appears to have misinterpreted their real meaning, and in consequence to have failed to draw the proper conclusion. But the results obtained and the conclusions drawn are far too important to permit this being done without comment; for, correctly interpreted, the results obtained by Mr. Cowell in this investigation afford most valuable information concerning the most difficult parts of the lunar theory. It is also desirable to show exactly how far the results deduced by Mr. Cowell are in accord with the observations, and to what extent and in what manner they may require supplementing, before they adequately represent the observed errors of the tables.

I have already drawn Mr. Cowell's attention to the fact that the coefficients yielded by his method of investigation are not the true coefficients of the argument being investigated, but "apparent" coefficients involving fractional portions of the coefficients of other arguments besides that being specially considered, and that unless precautions were taken to eliminate these disturbing elements, the results obtained by means of these apparent coefficients might prove most misleading (*Monthly Notices*, 1904 May, pp. 604–5).

Mr. Cowell does not seem, however, to have fully realised the necessity of taking these precautions, and has thought it sufficient when investigating the coefficient of the term with the arguments

$$[g \cdot g']$$

to take into account the influence exerted on the values obtained by errors in the coefficients of the associated terms with the arguments

$$[g \cdot g'] \pm \mathrm{D}$$

excepting in a few instances involving terms due to the disturbing action of the planets.

But this is not sufficient. If

$$\frac{1}{n}\sum_{1}^{n} \sin i\mathrm{D} = s_i \qquad \frac{1}{n}\sum_{1}^{n} \cos i\mathrm{D} = c_i$$

represent the mean value of the sine and cosine of the argument iD for the observations involved, it has been found from a discussion of the entire series of 150 years' observations, that with a considerable degree of approximation it may be taken that

$$s_1 = +0''.061 \quad s_2 = -0''.050 \quad s_3 = +0''.025$$
$$c_1 = -0''.525 \quad c_2 = -0''.111 \quad c_3 = -0''.175$$

so that terms depending on the associated arguments

$$\begin{array}{ccc} & \sin\{[g\,.\,g']\pm 2\mathrm{D}\} & \sin\{[g\,g']\pm 3\mathrm{D}\} \\ \cos\{[g\,g']\pm\mathrm{D}\} & \cos\{[g\,.\,g']\pm 2\mathrm{D}\} & \cos\{[g\,g']\pm 3\mathrm{D}\} \end{array}$$

will also cause small systematic errors in the apparent values of the coefficients of the argument

$$[g\,.\,g']$$

in addition to the larger discordances arising from the terms

$$\sin\{[g\,.\,g']\pm\mathrm{D}\}$$

which alone have been taken into account by Mr. Cowell.

But a more important case is the mutual systematic interference of the coefficients of the mean anomalies of the Moon and Sun. Let the terms depending on these arguments be denoted by

$$\mathrm{A}'\sin g+\mathrm{A}''\cos g+\mathrm{M}'\sin g'+\mathrm{M}''\cos g'$$

and let the apparent coefficients of these arguments obtained by Mr. Cowell's method be denoted by

$$\mathrm{A}_s\ \mathrm{A}_c\ \mathrm{M}_s\ \mathrm{M}_c$$

Then, as

$$g=g'-(\omega-\omega')+\mathrm{D}\qquad g'=g+(\omega-\omega')-\mathrm{D}$$

and denoting the mean value of the cosine of D by c_1, the values actually obtained for the apparent coefficients, by Mr. Cowell's method, will be

$$\begin{aligned} \mathrm{A}_s &= \mathrm{A}'-c_1\{\mathrm{M}'\cos(\omega-\omega')-\mathrm{M}''\sin(\omega-\omega')\} \\ \mathrm{A}_c &= \mathrm{A}''-c_1\{\mathrm{M}'\sin(\omega-\omega')+\mathrm{M}''\cos(\omega-\omega')\} \\ \mathrm{M}_s &= \mathrm{M}'-c_1\{\mathrm{A}'\cos(\omega-\omega')+\mathrm{A}''\sin(\omega-\omega')\} \\ \mathrm{M}_c &= \mathrm{M}''+c_1\{\mathrm{A}'\sin(\omega-\omega')-\mathrm{A}''\cos(\omega-\omega')\} \end{aligned}$$

As the argument

$$\omega-\omega'=\varpi-\varpi'$$

has a period of nine years, it is obvious that there will be set up nine-year inequalities in the apparent values of the coefficients of both g and g', which may be very sensible in the mean values deduced from the results of twelve to twenty-five years' observations, though usually they will be small in the case of values deduced from series of observations extending over forty to fifty years.

It is obvious that similar but smaller inequalities will be set up by the coefficients of the term

$$g'\pm i\mathrm{D}$$

and similar inequalities of only half the period by the coefficients of the arguments

$$g-2g'\pm i\mathrm{D}.$$

When dealing with the mean values of coefficients derived from series of observations extending over fifty years, these inequalities will have little influence on the results obtained, and under these conditions they may be ignored, as has been done by Mr. Cowell.

But there is a further development to be considered.

Let

$$\mathrm{Z}'\sin(g+\zeta)+\mathrm{Z}''\cos(g+\zeta)$$
$$\mathrm{Y}'\sin(g'+\zeta')+\mathrm{Y}''\cos(g'+\zeta')$$

denote pairs of terms whose arguments only differ from g and g' by the arguments of long-period ζ and ζ' respectively. Then the values which will be deduced from observation will be

$$\begin{aligned}
\mathrm{A}_s = \mathrm{A}' &+\mathrm{Z}'\cos\zeta - c_1\{\mathrm{M}'\cos(\omega-\omega')+\mathrm{Y}'\cos(\omega-\omega'+\zeta')\}\\
&-\mathrm{Z}''\sin\zeta + c_1\{\mathrm{M}''\sin(\omega-\omega')+\mathrm{Y}''\sin(\omega-\omega'+\zeta')\}\\
\mathrm{A}_c = \mathrm{A}'' &+\mathrm{Z}'\sin\zeta - c_1\{\mathrm{M}'\sin(\omega-\omega')+\mathrm{Y}'\sin(\omega-\omega'+\zeta')\}\\
&+\mathrm{Z}''\cos\zeta - c_1\{\mathrm{M}''\cos(\omega-\omega')-\mathrm{Y}''\cos(\omega-\omega'+\zeta')\}\\
\mathrm{M}_s = \mathrm{M}' &+\mathrm{Y}'\cos\zeta' - c_1\{\mathrm{A}'\cos(\omega-\omega')+\mathrm{Z}'\cos(\omega-\omega'+\zeta)\}\\
&-\mathrm{Y}''\sin\zeta' - c_1\{\mathrm{A}''\sin(\omega-\omega')-\mathrm{Z}''\sin(\omega-\omega'+\zeta)\}\\
\mathrm{M}_c = \mathrm{M}'' &+\mathrm{Y}'\sin\zeta' + c_1\{\mathrm{A}'\sin(\omega-\omega')+\mathrm{Z}'\sin(\omega-\omega'+\zeta)\}\\
&+\mathrm{Y}''\cos\zeta' - c_1\{\mathrm{A}''\cos(\omega-\omega')+\mathrm{Z}''\cos(\omega-\omega'+\zeta)\}
\end{aligned}$$

Now reckoning in years Y from the epoch Y_0, when the argument was zero, we may write

$$\omega-\omega' = \varpi-\varpi' = +40^\circ\cdot67\,(\mathrm{Y}-\mathrm{Y}_0)$$

so that the annual increment of the argument is rather less than 41°, and the period rather less than nine years. Hence, if the arguments denoted by ζ and ζ' be supposed to have annual increments differing from that of $(\omega-\omega')$ by not more than 5°, they will give rise to apparent inequalities in the values of the coefficients A_s, A_c and M_s, M_c of more than seventy years' period, and these will not disappear from the mean results derived from fifty years' observation.

These Mr. Cowell has failed to take into account.

There exist a considerable number of terms of this character in the expression for the Moon's longitude ; as, for instance, the term to which Mr. Cowell assigns the value

$$-0''\cdot70\sin(2\mathrm{D}-g+3\mathrm{V}-3\mathrm{E})$$

which has an argument whose annual increment differs from that of g by

$$2\varpi + 3V - 5E = 36^\circ{\cdot}92$$

and will give rise to apparent inequalities of ninety-seven years' period in the values found for the coefficient of g'. Another is the term to which M. Radau assigns the value

$$+0''{\cdot}181 \sin \{3V - 4E + 88^\circ\}$$

which will give rise to an inequality of similar period in the apparent value of the coefficient of the mean anomaly g, which will not disappear from the mean of fifty years' observations.

As the long-period inequalities in the Moon's mean longitude must give rise to subsidiary terms of the form

$$Z'' \cos (g + \zeta)$$

they must all produce inequalities of about nine years' period in the apparent value of the coefficients of g', and so systematically affect the values derived, from observation by Mr. Cowell, for the coefficients of such arguments as

$$g' \pm (\omega - \omega')$$

and these in turn affect the value he assigns to the real coefficient of the arguments

$$D \pm g'(\omega - \omega')$$

In the same manner inequalities of long period in the apparent coefficient of g' will give rise to inequalities of about nine years' period in the coefficients of g derived from observation, and so systematically affect the values found by Mr. Cowell for the coefficients of the arguments of the form

$$g \pm (\omega - \omega')$$
$$D \pm g \pm (\omega - \omega')$$

Mr. Cowell has not investigated the variations in the values of the coefficients of the argument g' derived from the observations, though they are of considerable magnitude and indicate the existence of inequalities of some importance. Therefore to supply the omission I take from my own results the following values of the coefficients of the expressions

$$M = +\Delta M_s + Q'_s \cos (\omega - \omega') + Q''_s \sin (\omega - \omega')$$
$$M_c = +\Delta M_c + Q'_c \sin (\omega - \omega') + Q''_c \cos (\omega - \omega')$$

derived from yearly values of the coefficients of the Sun's mean anomaly obtained through the solution of a system of simultaneous equations which serve to eliminate these fictitious inequalities. The value for each is derived from a group of

eighteen years' observations, so that the values of ΔM_s and ΔM_c are independent of the effects of the errors in Hansen's tabular coefficients, and represent the effects of omitted terms of very long period.

The values for these coefficients are

1891	$\Delta M_s = -$ ·60	$Q'_s = +$ ·61	$Q''_s = +$·12	$\Delta M_c = -$ ·41	$Q'_c = -$ ·47	$Q''_c = +$ ·03
82	= − ·83	= + ·75	= +·30	= − ·42	= − ·69	= + ·28
73	= − ·65	= + ·83	= +·29	= − ·35	= − ·58	= + ·08
64	= − ·44	= + ·66	= +·19	= − ·49	= − ·45	= + ·30
55	= − ·28	= + ·53	= +·03	= − ·32	= − ·33	= + ·53
46	= + ·03	= + ·97	= −·17	= + ·00	= − ·47	= + ·27
37	= − ·14	= + ·95	= −·20	= + ·04	= −1·28	= + ·84
28	= −1·04	= + ·98	= −·15	= + ·30	= −1·66	= + ·49
19	= − ·72	= +1·18	= −·31	= + ·14	= −1·85	= − ·34
10	= + ·30	= + ·82	= −·52	= − ·29	= − ·87	= + ·55
1801	= + ·51	= − ·04	= −·32	= − ·50	= − ·03	= + ·71
1792	= − ·44	= − ·40	= −·51	= − ·50	= + ·15	= + ·74
83	= − ·94	= + ·06	= +·11	= − ·49	= + ·43	= +1·02
74	= − ·36	= + ·02	= +·41	= −1·24	= − ·37	= + ·02
65	= − ·31	= + ·91	= +·37	= −1·05	= −1·23	= − ·24
1759	= − ·22	= +1·01	= +·42	= − ·29	= −1·65	= − ·09

These values show clearly the existence of inequalities of long period in the coefficients of g' which must give rise to apparent fictitious inequalities of about nine years' period in the values found by Mr. Cowell for the coefficients of the Moon's mean anomaly, whilst the large variations in the Q coefficients show the existence of important inequalities with arguments of similar period to that of the argument $(\omega - \omega')$, and these will give rise to fictitious inequalities of very long period in the values which Mr. Cowell has deduced for the coefficients of g, which will not disappear from the mean of fifty years' observation.

Similar terms, though of smaller dimensions, will arise depending on the argument

$$2\omega - 2\omega'$$

and must be taken into account before the true value of the *elliptic inequality* can be deduced by the method adopted by Mr. Cowell.

The motion of the lunar perigee will give rise to a further set of corrections which must be taken into account before the true value can be obtained of the corrections to the coefficients of the perturbations.

Writing for brevity

$$p = \omega - \omega' = \varpi - \varpi'$$

it can be shown that the terms

$$\begin{aligned} X' \sin x + X'' \cos x + C_1 \sin \{x - [ig' - jp]\} + C_2 \sin \{x - [ig' + jp]\} \\ + C_3 \sin \{x + [ig' - jp]\} + C_4 \sin \{x + [ig' + jp]\} \end{aligned}$$

will yield for the apparent coefficients of the argument x the values

$$X_s = X' - \frac{j}{i}\{m_1(C_1 - C_2 + C_3 - C_4) + m_2(C_1 + C_2 + C_3 + C_4)\} \cos j(p)$$

$$X_c = X'' - \frac{j}{i}\{m_1(C_1 + C_2 - C_3 - C_4) + m_2(C_1 - C_2 - C_3 + C_4)\} \sin j(p)$$

where approximately

$$m_1 = +0{\cdot}11 \qquad m_2 = +0{\cdot}01$$

The second of these coefficients is too small for the terms depending on it to be sensible, except for errors of very large dimensions, such as some of those in the coefficients of the perturbations adopted in Airy's lunar tables.

It was on account of the necessity of eliminating these fictitious inequalities and avoiding the many complex consequent corrections arising from unknown as well as known errors in the tabular values required to obtain the true values of the coefficients, that in my own investigations the values of the different coefficients were deduced in the form

$$\begin{aligned} C_s = C_i + \Sigma_1^i \{a_i' A^{i'} + a_i'' A_i'' + m_i' M_i' + m_i'' M_i'' + d_i' D_i' \\ + d_i'' D_i'' + \ldots\} \end{aligned}$$

The numerical factors $a_i' \ldots d_i'$ in these expressions range generally between $\pm 0{\cdot}25$, although d_1 is much larger, and the notation is

$$A_i' \sin ig + A_i'' \cos ig$$

By treating these expressions as a set of simultaneous equations, the values of all the subsidiary coefficients

$$A'A'' \ldots D_i'D_i''$$

were eliminated by the value for that year. In this manner not only are more accurate values obtained for the coefficient, but all these subsidiary terms and associated corrections are removed with the corresponding principal term.

(2) The first step in the investigations of the origin of the observed errors of the lunar tables is to consider the bearing of the results obtained from theory by M. Radau (*Annales de*

l'Observatoire de Paris, Mémoires, tome xxi., B. 1, 1895). Adopting the Delaunay-Hill method, M. Radau has calculated from theory the values of the perturbations produced by the disturbing action of the planets in the expression for the longitude of the Moon. Assuming that the method employed yields the complete value of the coefficients of the terms arising from the disturbing action of the planets, a critical examination will show that the care and completeness with which the calculations have been made, in the great majority of cases must ensure the results obtained representing the true theoretical coefficient of the terms within a few thousandths of a second of arc. This being so, to take into account the complete theoretical value of the perturbations produced by the planets it should suffice to supplement the tables by the small number of terms found to possess sensible coefficients by the calculations of M. Radau.

Adopting M. Radau's results as expressing the complete effect of the disturbing action of the planets, by omitting the terms reflecting the secondary effects of the terms of very long period, the terms required to express the complete perturbations in the Moon's longitude due to the action of the planets may be divided into two classes—the terms whose coefficients exceed a quarter of a second of arc and the terms whose coefficients are greater than a tenth of a second of arc but less than one quarter of a second; the terms with still smaller coefficients being disregarded as insensible.

These two classes are :—

	Argument.	Coefficient according to M. Radau.	Coefficient adopted by Hansen.
		″	″
Class I.	sin {2D − *g* + 2E − 2J }	−0·881	
	sin { V − E }	−0·860	−1·10
	sin {2D − *g* + 3V − 3E }	−0·681	...
	sin { E − J }	+0·646	+ ·74
	sin { E − 2M − 49}	+0·422	+ ·32
	sin { 2V − 3E + 85}	−0·348	
	sin {2D − *g* + 2E − 3J + 7}	+0·316	
	sin { 2V − 2E }	+0·283	+ ·43
Class II.	sin { 2E − 2M }	+0·228	+ ·24
	sin {2D − *g* − E + J }	−0·220	
	sin {2D − 2*g* + 2E − 2J }	−0·206	
	sin { 2E − 2J }	−0·196	− ·24
	sin {2D − 2*g* + 2E − 3J }	+0·194	
	sin {2D − *g* + 2V − 2E }	+0·192	
	sin { 3V − 4E + 88}	−0·181	
	sin {2D − E + J }	−0·175	
	sin { E − 2J + 298}	+0 172	

	Argument.	Coefficient according to M. Radau.	Coefficient adopted by Hansen.
		"	"
Class II.	$\sin\{2D-g-2V+2E\ \ \ \

Originally this may have been done to show that the values deduced from theory by M. Radau agreed with the observed values; and in this sense Mr. Cowell appears to have regarded the results deduced by him from the observations, taking them as showing that Hansen's values when corrected yielded the values

$$\begin{aligned}
\{-1''\cdot10-0''\cdot34=-0''\cdot76\}\ &\sin\{\ V-\ E\ \ \ \ \ \ \}\\
\{+0\cdot43-0\cdot10=+\ \cdot33\}\ &\sin\{2V-2E\ \ \ \ \ \}\\
\{+0\cdot00-0\cdot29=-\ \cdot29\}\ &\sin\{2V-3E+85^\circ\}
\end{aligned}$$

which were in far better agreement with the theoretical values deduced by M. Radau than with those adopted by Hansen. At the same time his results indicated that Hansen's value of the term with the argument (E—J) agreed far better with the observations than that deduced by M. Radau, and that the value indicated by the observations for the term with the argument (E—2M—49°) agreed with neither Hansen nor Radau.

Finally the results given in the *Monthly Notices* for 1904 December appear to indicate that Mr. Cowell had found it desirable to throw over the values deduced from theory by M. Radau, as in his adopted values he changes very materially the coefficients yielded by theory.

Thus he makes the changes

$-0''\cdot881\sin\{2D-g+2E-2J\ \ \ \ \ \ \}$	coefficient increased to		$-1''\cdot10$
$+0\cdot316\sin\{2D-g+2E-3J+7^\circ\}$	„	„	$+0\cdot40$
$-0\cdot167\sin\{\ \ \ g-2E+2J\ \ \ \ \ \}$	„	„	$-0\cdot30$
$+0\cdot101\sin\{2D-g+\ E-\ J\ \ \ \ \}$	„	„	$+0\cdot20$
$-0''\cdot860\sin\{\ \ \ \ \ \ V-\ E\ \ \ \ \ \ \}$	coefficient reduced to		$-0''\cdot80$
$+0\cdot192\sin\{2D-g+2V-2E\ \ \ \ \}$	„	„	$+0\cdot00$
$-0\cdot181\sin\{\ \ \ \ \ \ 3V-4E+88^\circ\}$	„	„	$-0\cdot10$
$-0\cdot172\sin\{2D-g+2V-2E\ \ \ \ \}$	„	„	$-0\cdot10$

These are very material changes in the values for the coefficients of these terms, and the modified values adopted by Mr. Cowell as indicated by the observations are quite inconsistent with the results deduced from theory by M. Radau.

Mr. Cowell appears to regard these alterations he has made as simply changes from one theoretical value to another just as permissible, and merely as indications that the value deduced from theory by M. Radau is more accurate than that similarly deduced by Hansen and employed in his tables, or *vice versa*. But it means much more. For the value for the coefficients of these terms employed by Hansen consists merely of the principal term arising from the indirect action of the planet on the Moon

through the perturbations it produces in the motion of the Earth. It forms a rough approximation to the true value, and cannot be compared with the more accurate and complete result deduced by M. Radau, which corresponds with the result which would have been obtained by Hansen had he thought it necessary to carry his calculations to the same degree of approximation.

But the changes proposed by Mr. Cowell are entirely inconsistent with M. Radau's values, which are, speaking generally, accurate within a few thousandths of a second of arc, and cannot possibly be changed by 20 or 30 per cent. of their values on any permissible assumption except that the method of calculation employed by M. Radau fails to give the complete values of the terms. Thus it is possible that a still more detailed calculation of the value of the coefficient of the Jovian evection by the same method as employed by M. Radau might yield a slightly corrected value ; but it would be one lying between the limits of $0''{\cdot}90$ and $0''{\cdot}88$; and no such revision or elaboration of the details of the calculation could raise the value to over $1''{\cdot}00$, much less to $1''{\cdot}10$, the value adopted by Mr. Cowell. Such a modification is only possible by supposing that the Delaunay-Hill method adopted by M. Radau fails to give the complete value of the coefficients of these terms owing to the omission of a class of important factors necessary to the complete value of the coefficients.

If the comparison of the observed and tabular places of the Moon had shown that the values, calculated from theory, of the coefficients of the more important terms arising from the disturbing action of the planets were in complete accord with those indicated by observation, and that no further correction to the adopted theoretical values were left outstanding, then it would have sufficed to have taken into account such of the terms which the theoretical calculations indicated were large enough to be sensible, and to have neglected all the others as insensible.

But as the observations show that these values deduced from theory are not in accord with the observations, but need supplementing in the manner indicated by Mr. Cowell (*Monthly Notices*, 1904 December, Table X.), thus indicating that the method adopted for calculating these terms cannot yield their complete value, then, in investigating the origin of the errors of the lunar tables, it is essential to take into consideration, not only the possible errors in the values of the larger coefficients of the term adopted from theory, but also the possible errors in the terms having small coefficients according to the defective theory.

In this respect Mr. Cowell's investigation fails, for he has confined his attention to such terms as have coefficients of over a tenth of a second (with one exception), and has omitted to pay any attention to the associated terms of similar origin whose coefficients are small according to M. Radau. Thus Mr. Cowell deduces corrections to M. Radau's value of the term

$$-0''{\cdot}681 \sin\{2\mathrm{D} - g + 3\mathrm{V} - 3\mathrm{E}\}$$

but ignores altogether the companion term

$$-0''\cdot019 \sin\{2D-g+4E-4M\}$$

though the period of the two terms is so close that without special care they cannot be separated by a discussion of the observations. But if the coefficient of the former is held to be incomplete, so must the coefficient of the latter, and therefore corrections to both must be deduced from the observations before either can be deemed accurate.

Similarly with the two terms to which M. Radau assigns the values

$$-0''\cdot181 \sin\{3V-4E+88^\circ\}$$

and

$$+0''\cdot055 \sin\{3E-4M+328^\circ\}$$

Mr. Cowell considers the first and deduces a correction, yet ignores the second, though the periods of the two are so very similar that for years their coefficients cannot be separately deduced from the observations.

(3) These essential preliminary considerations having been dealt with, it remains to consider the actual corrections to the lunar tables deduced by Mr. Cowell from his discussion of the observations.

Mr. Cowell's results may be divided into

I. His corrections to the secular terms and empirical inequalities of very long period.

II. The values deduced by him for the coefficients of the inequalities arising from the disturbing action of the planets.

III. The corrections he deduces for the tabular coefficients of the perturbation due to the direct action of the Sun.

These will be considered in order.

(4) Mr. Cowell's value for the corrections to Hansen's secular terms, and for the semi-secular terms of very long period which should be included in the tables, appears to have been originally

$$\begin{aligned}&-5''\cdot44-29''\cdot17(T-1800\cdot0)-3''\cdot76(T-1800\cdot0)^2\\&+17''\cdot86 \sin\{0^\circ\cdot99[Y-1850\cdot50]+146^\circ\cdot9\}\\&+15''\cdot34 \sin\{18V-16E-a+35^\circ\cdot3\}\end{aligned}$$

to which would have to be added the corrections necessary to reduce Hansen's tabular value for the two long *Venus* terms to their values according to the latest published theoretical calculation, which give

$$\begin{aligned}&+14''\cdot40 \sin\{18V-16E-a+30^\circ\cdot6\}\\&+0''\cdot27 \sin\{8V-13E+228^\circ\cdot0\}\end{aligned}$$

To these Mr. Cowell adds the two smaller empirical terms of long period

$$+2''\cdot35 \sin \{5^\circ\cdot44[Y-1850\cdot50]+55^\circ\}$$
$$+\ \cdot73 \sin \{8\cdot60[Y-1850\cdot50]+15\}$$

This may be called Cowell's Solution I. It corresponds to a value of the secular acceleration in the Moon's mean longitude of

$$+8''\cdot42(T-1800\cdot0)^2$$

At the same time Mr. Cowell points out that the above empirical term of 363 years' period can be replaced by the combination

$$+5''\cdot20-3''\cdot00(T-1800\cdot0)-4''\cdot40(T-1800\cdot0)^2$$
$$+12''\cdot16 \sin \{1^\circ\cdot10[Y-1850\cdot5]+152^\circ\cdot1\}$$

But this corresponds to a reduction of the secular acceleration of the mean motion to

$$+4''\cdot02(T-1800\cdot0)^2$$

or to only two thirds of the minimum theoretical value, and Mr. Cowell does not seem to refer further to this modification in his paper. It was distinguished as Cowell Solution II.* This solution assumes so great a reduction of the secular acceleration that it does not require further consideration.

Yet in his last paper in the *Monthly Notices* for 1905 January, p. 273, Mr. Cowell reverts to this alternative system of correction, but halves the proposed correction to the secular terms, adopting

$$+2''\cdot60-1''\cdot50(T-1800\cdot0)-2''\cdot20(T-1800\cdot0)^2$$

* I do not understand Mr. Cowell's remark in connection with this solution (*Monthly Notices*, 1904 November, pp. 34–5) where he says: "The above formula has been formed on the assumption that one, and only one, empirical term of very long period is to be introduced, and also the secular acceleration is not to be altered. With these limitations I believe the solution is unique; that is to say, it only admits of variations proportional to the errors of observation, and no totally different solution can be found." It appears to me to be a mere truism. The observed errors of the tables form a fixed simple curve. If no changes are to be made in the coefficients of the arguments derived from theory—for if so they would then become additional empirical terms which are barred—no change is to be made in the secular acceleration, and no change in the mean motion can represent a curve; all that is left to vary so as to represent the observed curve is the solitary empirical term, and of course there can be but one value of such a term which can represent the fixed curve of observation. Obviously no totally different value can do so. If it were meant that only one new empirical *argument* of long period was to be introduced, but that any desired changes might be introduced into the coefficients of any of the tabular terms of long period, then Mr. Cowell's solution being "unique" would be something of great importance. But it would not be a fact, for under these conditions by merely varying the coefficient of Hansen's direct *Venus* term it is possible to find a dozen empirical terms with periods from 250 to 400 years which will represent the observations between 1900 and 1750 quite as closely as Mr. Cowell's.

corresponding to the value of the secular acceleration of the mean motion of

$$+6''\cdot22(T-1800\cdot0)^2$$

which is very close to its theoretical value. Mr. Cowell does not state whether this halving of the secular terms is to be accompanied by a proportionate change in the empirical term of long period, which would bring it to the value

$$+15''\cdot0 \sin \{1^\circ\cdot05[Y-1850\cdot50]+351^\circ\cdot4\}$$

a term of some 343 years' period. Judging from Mr. Cowell's remark (*Monthly Notices*, 1904 November, p. 35), that the ancient eclipses seem to "point to a negative correction to the secular acceleration of about 2″, my conclusion is that if a single long-period empirical term alone exists, its period should be about 350 years." It would seem that this is the term he intends.

If so, his finally adopted correction, Cowell No. III., becomes Hansen

$$\begin{aligned}
&-2''\cdot84-30''\cdot67(T-1800\cdot0)\\
&\quad-5\cdot96(T-1800\cdot0)^2\\
&\quad+14\cdot40 \sin \{18V-16E-g+30^\circ\cdot6\}-15''\cdot34 \sin \{18V\\
&\qquad\qquad-16E-g+35^\circ\cdot3\}\\
&\quad+0\cdot27 \sin \{8V-13E+228^\circ\cdot0\}-21''\cdot47 \sin \{8V-13E\\
&\qquad\qquad+274^\circ\cdot4\}\\
&\quad+15\cdot00 \sin \{1^\circ\cdot05[Y-1850\cdot50]+351^\circ\cdot4\}\\
&\quad+2\cdot35 \sin \{5^\circ\cdot44[Y-1850\cdot50]+55^\circ\ \ \}\\
&\quad+\ \cdot73 \sin \{8^\circ\cdot60[Y-1850\cdot50]+15^\circ\ \ \}
\end{aligned}$$

If however the empirical term of Mr. Cowell's alternative solution is to be left unaltered when the secular corrections are halved, the modified expression will form Cowell Solution IV.

It remains to ascertain how far these systems of corrections deduced by Mr. Cowell are in accord with the observed errors of the tables.

For the purpose of comparison a set of standard tabular errors were formed as follows. The tabular error yielded by each observation was equated to the expression

$$\begin{aligned}
&L+M_s \sin g'+M_c \cos g'+A_s \sin g+A_c \cos g+P_s \sin D\\
&\qquad\qquad+P_c \cos D+V_s \sin 2D+V_c \cos 2D+sS
\end{aligned}$$

and the mean of all the observations for the year was taken. The small outstanding corrections depending on the *sine* and *cosine* of the *annual equation*, *mean anomaly*, *parallactic inequality*, and *variation*, together with the necessary correction

for semi-diameter, were then eliminated by their values determined from the observations of that year. This yielded the mean error for the year, sensibly free from all tabular errors except those of moderately long period. To eliminate these there was applied the system of correction for these terms deduced by Mr. Cowell from the observations and given in Table X. in the *Monthly Notices* for 1904 December. This system was taken as being

$$
\begin{aligned}
&-0''.59 \sin \{\ \omega - \omega'\ \}\\
&+0''.09 \sin \{2\omega - 2\omega'\}\\
&+0''.16 \sin \{2\omega\}\\
&-1''.15 \sin \{\ \&\ \}\\
&+0''.30 \sin \{\ V - E\ \}\\
&-0''.13 \sin \{2V - 2E\}\\
&-0''.30 \sin \{2V - 3E + 85^\circ\}\\
&+0''.08 \sin \{\ E - 2M - 49^\circ\}\\
&-0''.04 \sin \{\ E - J\ \}\\
&+0''.20 \sin \{\ E - 2J + 298^\circ\}\\
&+0''.20 \sin \{2\varpi - 2J\}
\end{aligned}
$$

The yearly values thus corrected were divided into nine-year groups, and the mean of each group taken as the standard tabular error for the middle of the group. Such errors were determined at different five-year epochs.

From the discordance between the observed and the calculated errors of Hansen's tables between the years 1862–1878 it has been shown that the outstanding errors of the tables are in the great majority of cases much below $\pm 2''.0$, and the probable error of an observation, apart from the errors of the tables themselves, is less than $\pm 1''.0$ (*Memoirs R.A.S.* xlviii. p. 392). As the mean error for each year generally depends on a hundred observations, the accidental error of the mean for the year should be less than a sixth of a second of arc, and the discordance between the observed and the properly corrected tabular places should rarely exceed $\pm 0''.30$ in the mean for the year. The mean error outstanding in a group of nine years should rarely exceed $\pm 0''.05$. Hence any discordance between the calculated and the observed errors of the tables exceeding $\pm 0''.25$ in the mean of a nine-year group must be regarded as due to imperfections in the adopted system of corrections, and should such discordances exceed $\pm 0''.50$ the adopted system must be regarded as a failure.

The earlier observations are not so accurate as those made with the Greenwich Transit Circle, but still those made between 1850–1817 with the Troughton Transit and Mural Circle, when properly corrected, are not much inferior to those made since 1851 with the Transit Circle. Probably the above values should be

increased by about a third, so that when properly corrected the tables should not show discordances between observation and calculation exceeding $\pm 0''\cdot40$ in the mean of any year, or $\pm 0''\cdot20$ in the mean of a nine-year group. Hence during this period the occurrence of discordances exceeding $\pm 0''\cdot60$ between the observed and calculated mean error for a nine-year group must be held as showing the system of correction adopted to break down. The still earlier observations made with the Bird Transit and Brass Quadrant are even more uncertain, and it may be taken that the observed discordance between observation and calculation should not reach $\pm 0''\cdot60$, and such discordances as $\pm 1''\cdot00$ be held as proof of failure.

The earlier observations prior to 1750 come into a different category. Yet judging from the discordances shown by separate observations, and allowing for the much smaller number of observations going to form a nine-year group, the discordance between observation and a proper system of tabular corrections should not exceed $\pm 1''\cdot00$ between 1750 and 1710, nor exceed $\pm 2''\cdot00$ between 1700 and 1660, but might rise to $\pm 4''\cdot00$ for the epochs prior to 1660.

Hence it may be laid down that no system of corrections to the tables can be regarded as satisfactory if the discordance between the observed and calculated errors of the tables exceed the following amounts :—

Period	1900–1851	Maximum discordance	$=\pm$ ″·50
	1850–1820		$=\pm$ ·75
	1820–1750		$=\pm 1''\cdot00$
	1750–1700		$=\pm 1''\cdot50$
	1700–1660		$=\pm 2''\cdot50$
	1660–1640		$=\pm 4''\cdot00$
	1640–1620		$=\pm 6''\cdot00$

whilst a proper system of corrections should agree with the observed error to within two-thirds of these values. It will be seen that the possibility of systematic error has been magnified as much as possible in forming the preceding estimate so as to form a criterion as favourable as possible to the system of adopted corrections.

Then the comparison of the three systems of corrections deduced by Mr. Cowell, and distinguished as "Cowell Solution I.," "Cowell Solution III.," and "Cowell Solution IV." (omitting No. II. owing to the impossible value of the secular acceleration), with the system of standard errors obtained as described above, yields the following values for the outstanding mean error of the tables at equidistant five-year epochs from 1900 to 1630. (The observed error for the epoch 1900 is derived from a group of only five years, as the observed errors for the years 1903 and 1904 were not available.)

	Observed Tabular Error.	(Observed—Computed) Tabular Error. Cowell Solution I.	Cowell Solution III.	Cowell Solution IV.	Cowell Solution IV.*
	″	″	″	″	″
1900	+25·61	+ 1·17	− ·23	− ·56	+ ·94
1895	+20·57	− 1·29	− 2·66	− 3·23	− 1·73
90	+17·48	− ·74	− 2·03	− 2·79	− 1·29
85	+14·99	− ·26	− 1·50	− 2·47	− ·97
80	+11·99	+ ·33	− ·83	− 2·01	− ·51
75	+ 8·54	+ ·58	− ·53	− 1·88	− ·38
70	+ 5·51	+ 1·64	+ ·57	− ·97	+ ·53
65	+ 1·70	+ 1·04	+ ·04	− 1·65	− ·15
60	− 1·54	+ ·33	− ·62	− 2·47	− ·97
55	− 2·09	+ ·88	− ·01	− 2·00	− ·50
50	− ·83	+ 2·27	+ 1·44	− ·66	+ ·84
45	− ·26	+ 1·92	+ 1·15	− 1·10	+ ·39
40	+ ·10	+ 1·17	+ ·47	− 1·88	− ·38
35	+ 1·55	+ 1·51	+ ·96	− 1·49	+ ·01
30	+ 1·19	+ 1·97	+ ·18	− 1·36	+ ·14
25	+ 1·08	+ ·52	− ·02	− 2·64	− 1·14
20	+ 2·65	+ 2·72	+ 1·74	− ·95	+ ·55
15	+ 3·23	+ 3·98	+ 2·57	− ·18	+ 1·32
10	+ 1·94	+ 1·79	+ 1·43	− 1·35	+ ·15
05	+ 2·23	+ 2·17	+ 1·87	− ·94	+ ·56
1800	+ 2·82	+ 2·82	+ 2·60	− ·24	+ 1·26
1795	+ 1·91	+ 2·24	+ 2·02	− ·86	+ ·64
90	+ 1·16	+ 1·57	+ 1·44	− 1·43	+ ·07
85	+ 1·68	+ 2·33	+ 2·27	− ·57	+ ·93
80	+ 1·44	+ 1·72	+ 1·71	− 1·01	+ ·49
75	+ 1·79	+ 1·64	+ 1·69	− 1·10	+ ·40
70	+ 2·92	+ 1·67	+ 1·80	− ·94	+ ·56
65	+ 2·68	+ ·97	+ 1·17	− 1·51	− ·01
60	+ 1·77	− ·24	− ·04	− 2·69	− 1·19
55	+ ·25	+ ·97	+ ·37	− 2·18	− ·68
50	+ ·31	+ ·77	+ ·28	− 1·35	+ ·15
45	− 3·05	+ ·41	+ ·81	− 1·54	− ·04
40	− 5·08	+ 1·58	+ 2·04	− ·62	+ ·88
35	− 5·62	+ 4·35	+ 4·87	+ 2·73	+ 4·23
30	− 6·43	+ 6·12	+ 6·70	+ 4·71	+ 6·21
25	− 8·04	+ 6·64	+ 7·28	+ 5·43	+ 6·93
20	− 9·95	+ 6·14	+ 6·84	+ 5·16	+ 6·66
15	−10·45	+ 6·88	+ 7·53	+ 6·10	+ 7·60

	Observed Tabular Error.	(Observed—Computed) Tabular Error. Cowell Solution I.	Cowell Solution III.	Cowell Solution IV.	Cowell Solution IV.*
	″	″	″	″	″
1710	−13·80	+ 4·67	+ 5·49	+ 4·14	+ 5·64
05	−18·80	+ 1·17	+ 2·03	+ ·86	+ 2·36
1700	−21·67	+ ·09	+ 1·00	+ ·05	+ 1·55
1695	−23·95	− ·41	+ ·58	− ·17	+ 1·33
90	−28·35	− 2·69	− 1·66	− 2·22	− ·72
85	−31·48	− 4·13	− 3·04	− 3·36	− 1·86
80	−31·62	− 2·17	− 1·05	− 1·15	+ ·35
75	−32·74	− 1·40	− ·34	− ·09	+ 1·41
70	−33·80	− ·16	+ 1·05	+ 1·44	+ 2·94
65	−35·73	− ·20	+ 1·03	+ 1·65	+ 3·15
60	−36·90	+ ·49	+ 1·76	+ 2·65	+ 4·15
55	−37·35	+ ·74	+ 3·01	+ 3·13	+ 4·63
50	−40·48	− 2·26	− ·03	+ ·36	+ 1·86
45	−43·25	− 6·40	− 5·13	− 3·47	− 1·97
40	−42·76	− 7·57	− 6·31	− 4·49	− 2·90
35	−43·20	− 10·25	− 9·03	− 6·87	− 5·37
1630	−47·94	−16·66	−15·54	−13·09	−11·59

Tested by this criterion which has been laid down, Solution No. I. fails entirely even for the modern observations between 1900 and 1850, and the greater number of the residuals are far larger than is allowable with any permissible solution throughout the entire period 1900–1750. Diminishing the calculated values by a second of arc improves the solution, but still leaves the outstanding residuals far too large during the greater part of the period 1900–1750, the discordances ranging between $+2''\cdot98$ and $-2''\cdot29$, instead of never exceeding a second of arc. When the comparison is carried into the earlier epochs before 1750, the calculated values fail altogether adequately to represent the observations, and are quite discordant with the results obtained from the numerous occultations of stars during the periods 1740–1710 and 1690–1670.

Solution No. III. breaks down completely as thus tested, not only for the earlier observations, but likewise for the modern ones, the discordances between the calculated and observed errors ranging from $+2''\cdot60$ in 1800 to $-2''\cdot66$ in 1895 ; whilst they run up to over 6″ between 1730 and 1710.

Solution No. IV. evidently requires a further correction to the epoch, and the calculated values should be decreased by about $1''\cdot5$. As thus modified the values are treated as Solution IV.* This modified solution represents the more modern observed errors of the tables with greater closeness than either of the others, as the discordances range only between $+1''\cdot32$ in

1815 to $-1''\cdot73$ in 1895. These are still far too large, however, and like the other solutions it fails completely to represent the earlier occultations of stars, the discordance rising to over $6''$ between 1735 and 1710.

It is not surprising that Mr. Cowell's solutions fail to represent the earlier observations, for he has deduced his values solely from the Greenwich observations between 1900 and 1750. But this century and a half is quite insufficient to determine properly the values of inequalities with periods of from 300 to 400 years. For it is possible to find small portions of the arc of a number of terms with very variable parameters and periods which will represent the very simple fixed observed curve of error. And this is rendered still easier by the assumption of suitable empirical terms of shorter period which will serve to bring the radius of curvature at the middle and ends of the adopted curve into better accord with the curve of observed tabular error. But though this shorter portion of these different curves may thus be brought roughly into accord with the observed curve of tabular error during the century and a half between 1900 and 1750, there is no security that the remaining portion forming the larger half of the complete curve shall not be utterly discordant with each other and run altogether away from the curve of observed tabular error. It is only when the observations of the century prior to 1750 are taken into account that it becomes possible to properly test the manner in which the adopted term of very long period represents the observations, as it enables the comparison to be extended over the whole, or nearly the whole, arc of curvature.

A critical examination of the manner in which the observed values are represented by the separate constituents of Mr. Cowell's empirical solutions will show that the observations prior to 1750 cannot be satisfied by any term with a period much more than 300 years, and that his corrections would be improved by bringing the period of the principal empirical term nearer to 300 years and slightly shifting forward the epochs. And there can be little doubt that Mr. Cowell's empirical argument

$$\sin\{1^\circ\cdot10(Y-1850\cdot50)+152^\circ\cdot1\}$$

is identical with the argument shown by theory to exist in the expression for the Moon's longitude with the form

$$\sin\{3V-7E+4M+326^\circ\cdot8\}$$

which can be written as

$$\sin\{1^\circ\cdot19(Y-1850\cdot50)+153^\circ\cdot3\}$$

Only Mr. Cowell's coefficient is much greater than that assigned by theory.

Similarly an examination of the way in which the observed

values are represented by Mr. Cowell's two auxiliary terms of shorter period will show that the full coefficients are only required by the observations between 1870 and 1780, much smaller values being indicated by the observations at the ends of the series of a hundred and fifty years' observations, whilst the observations for the period prior to 1730 would be far better represented by changing the signs of the coefficients. In fact, the terms would much better represent the observation if their coefficients were written in the form

$$+2''\cdot35 \cos \{1^\circ\cdot2[Y-1850\cdot5]+30^\circ\}$$
$$+\ \cdot73 \cos \{1^\circ\cdot2[Y-1850\cdot5]+20\}$$

This would indicate that the terms were really compound terms due to the apparent coalescence of pairs of terms of similar period with the arguments of annual increment

$$5^\circ\cdot44-1^\circ\cdot20 = 4^\circ\cdot24$$
$$5\cdot44+1\cdot20 = 6\cdot44$$

and

$$8\cdot60-1\cdot20 = 7\cdot40$$
$$8\cdot60+1\cdot20 = 9\cdot80$$

and should be written in the form

$$+2''\cdot35\{\tfrac{1}{2}\cdot a_1 \sin(4^\circ\cdot24[Y-1850\cdot5]+220^\circ) + \tfrac{1}{2}b_1 \sin(6^\circ\cdot44[Y-1850\cdot5]+250^\circ)\}$$
$$+0''\cdot73\{\tfrac{1}{2}\cdot a_2 \sin(7^\circ\cdot40[Y-1850\cdot5]+205^\circ) + \tfrac{1}{2}b_2 \sin(9^\circ\cdot80[Y-1850\cdot5]+185^\circ)\}$$

where a_1 b_1, a_2 b_2 are numerical fractions differing little from unity.

(5) The next matter to be considered is the manner in which Mr. Cowell has dealt with the coefficients of the terms of shorter period whose values he has deduced from the observations made at Greenwich between 1900 and 1750. I do not think that he has done justice to his own results, especially in the case of terms arising from the disturbing action of the planets and that in his anxiety to reconcile his results with those derived from theory by M. Radau, he has often failed to realise the true meaning of the results he has obtained.

Mr. Cowell's three periods (i), (ii), (iii) depend on the observations of the periods 1750–1805, 1806–1851, and 1847–1901 respectively, each covering about fifty years, so that the probable errors of the values of the coefficients of his arguments should certainly not exceed $\pm0''\cdot05$ for period (i), nor $\pm0''\cdot04$ for period (ii), nor $\pm0''\cdot03$ for period (iii), and quantities twice as large as these probable errors cannot be passed over as due to accidental errors, but must be taken into account. Yet

Mr. Cowell frequently ignores much larger quantities than these as due to accidental errors, or as being unimportant, or at any rate as something which may be disregarded. This is especially so when they form the coefficient of a cosine term for which he fails to find any theoretical equivalent. But this is not doing justice to his own results, and is certainly not permissible. These coefficients must be shown to be due to the accumulation of accidental error, or to be the consequence of errors of observation or reduction, or to arise from the systematic influence of some other term. This must not be assumed.

When the observations yield as a correction to the tabular term

$$A \sin a$$

the terms

$$a' \sin a + a'' \cos a$$

unless a', the coefficient of the sine of the argument, is at least twice as large as a'', the coefficient of the cosine term, it cannot be used as a real quantity to be applied as a correction to the tabular coefficient A, but must be considered a quantity with a probable uncertainty at least as great as a''. For if the observed values are affected by errors of such magnitude or character that the coefficient a'' can be regarded as due to them, then the coefficient a' must be held to be equally liable to them and as uncertain in consequence by at least the amount of a''. On the other hand, if the coefficient a'' be held to be due to some small associated term of similar period, then a' will be just as likely to be similarly affected to the same extent. Yet Mr. Cowell frequently ignores this, and holds the sine coefficient as real and usable as a correction whilst the larger cosine coefficient is dismissed as fictitious. Thus, he deduces the following correction to the *evection* from the observations of the period 1847–1901

$$+0''{\cdot}03 \sin (2D-g) - 0''{\cdot}31 \cos (2D-g)$$

and says : "The observed and theoretical *evection* are in close accordance. This disposes of a correction of Hansen's" (*Monthly Notices*, 1904 December; p. 148). Yet how can this be justified in the face of the *cosine* term with a coefficient of $-0''{\cdot}31$? If the coefficient of the *cosine* term is due to the effects of errors of observation or reduction, then the coefficient of the *sine* term must be held to be equally uncertain by $\pm 0''{\cdot}31$; and if the coefficient of the cosine term be held due to some associated term of analogous period, then equally this associated term may similarly affect the sine term. In either case the true value of the coefficient of the sine term may have any value within $\pm 0''{\cdot}40$, and cannot be held to show that the theoretical and observed values are in close accord. Nor is it possible to dispose of the large coefficient of the cosine term by ascribing it to an error in tabular place of the *perigee*, because, if so, the coefficient

of the cosine of the mean anomaly should be five times as large and of the opposite sign, whereas it is only half as great and of the same sign.

This tendency to ignore inconvenient factors is very noticeable in the case of the terms due to the disturbing action of the planets when the value of the coefficients deduced from the observations is inconsistent with the theoretical values calculated by M. Radau. Mr. Cowell, then, often fails to do justice to his own results, and so has overlooked most important deductions.

Thus, in the case of the term to which M. Radau assigns the value

$$-0''\cdot348 \sin (2V-3E+85^\circ)$$

Mr. Cowell deduces the values

Period	(i)	$-''\cdot44 \sin (2V-3E+85^\circ)$	$+''\cdot05 \cos (2V-3E+85^\circ)$
	(ii)	$-\cdot40 \sin (\quad\;\; ,, \quad\;\;)$	$-\cdot13 \cos (\quad\;\; ,, \quad\;\;)$
	(iii)	$-\cdot24 \sin (\quad\;\; ,, \quad\;\;)$	$+\cdot22 \cos (\quad\;\; ,, \quad\;\;)$

and adopts the value

$$-0''\cdot30 \sin (2V-3E+85^\circ)$$

leaving the residuals

(i)	$-''\cdot14 \sin (2V-3E+85^\circ)$	$+''\cdot05 \cos (2V-3E+85^\circ)$
(ii)	$-\cdot10 \sin (\quad\;\; ,, \quad\;\;)$	$-\cdot13 \cos (\quad\;\; ,, \quad\;\;)$
(iii)	$+\cdot06 \sin (\quad\;\; ,, \quad\;\;)$	$+\cdot22 \cos (\quad\;\; ,, \quad\;\;)$

These are all much larger than is consistent with their being the results of mere errors of observation or reduction, and to ignore them is merely to neglect the evidence of the existence of other terms of long period than those considered by Mr. Cowell. For example, suppose these discordances due to an associated term differing in the annual increment of its argument by about $1^\circ\cdot5$, and having a coefficient of about $+0''\cdot30$ and an epoch near 1848. The elimination of the effects of such a term from the preceding residuals would leave them approximately as

(i)	$-''\cdot06 \sin (2V-3E+85^\circ)$	$+''\cdot33 \cos (2V-3E+85^\circ)$
(ii)	$-\cdot36 \sin (\quad\;\; ,, \quad\;\;)$	$+\cdot02 \cos (\quad\;\; ,, \quad\;\;)$
(iii)	$-\cdot17 \sin (\quad\;\; ,, \quad\;\;)$	$+\cdot03 \cos (\quad\;\; ,, \quad\;\;)$

and so show that the value of the coefficient deduced by Mr. Cowell, on allowing for the superior weight of the modern observations, should be increased by over $0''\cdot20$, or be brought up to

$$-0''\cdot50 \sin (2V-3E+85^\circ)$$

There is nothing forced in the assumption of such a term. For example, the term depending on the argument

$$g - 20V + 19E$$

is of the required character. And the very possibility of such a result shows that it is impossible to pass over such residuals as these and adopt the mean of the sine coefficient as indicating the true correction for the coefficient of the term under investigation.

It will be noticed that in the case just supposed there is left an outstanding residual of some dimensions in the values deduced from period (i). That suffices to show that the hypothetical correction assumed really fails to properly meet all the facts, and that the true correction is rather more complex than that assumed. But Mr. Cowell does not hesitate to leave far larger residuals. There is the case of the term arising from the disturbing action of *Jupiter* to which M. Radau assigns the value

$$-0''\cdot139 \sin (J + 99^\circ)$$

Mr. Cowell deduces from the observation the values

Period	(i)	$-0''\cdot50 \sin (J + 99^\circ) + 0''\cdot44 \cos (J + 99^\circ)$
	(ii)	$-\ \cdot19 \sin (\ \ ,,\ \ ) - \cdot15 \cos (\ \ ,,\ \ )$
	(iii)	$-\ \cdot14 \sin (\ \ ,,\ \ ) - \cdot06 \cos (\ \ ,,\ \ )$

and adopts the value

$$-0''\cdot10 \sin (J + 99^\circ)$$

leaving outstanding the residuals

(i)	$-0''\cdot40 \sin (J + 99^\circ) + 0''\cdot44 \cos (J + 99^\circ)$
(ii)	$-\ \cdot09 \sin (\ \ ,,\ \ ) - \cdot15 \cos (\ \ ,,\ \ )$
(iii)	$-\ \cdot04 \sin (\ \ ,,\ \ ) - \cdot06 \cos (\ \ ,,\ \ )$

But residuals of this character cannot be passed over in silence. The very large outstanding residuals left for the first period cannot be ascribed to mere imperfections in the earlier observations unless Mr. Cowell is prepared to throw over the whole of the results he has deduced from the observations of this period (i); for what value can be possessed by numerous corrections ranging between $+0''\cdot05$ and $+0''\cdot15$ if the observations are so imperfect that the values deduced are liable to discordances amounting to $\pm 0''\cdot40$? Obviously the true cause for these residuals is not imperfections in the observations, but that the values deduced from the coefficients of this particular argument are affected by systematic errors, probably due to the existence of two or more terms of similar period; and until the influence of

the disturbing causes is traced and eliminated, the value deduced for the coefficient must be deemed most uncertain.

As a matter of fact there are a number of terms giving rise to systematic variations in the apparent value of the coefficients of this term depending on the perturbations of *Jupiter*. For instance, the terms depending on the arguments

$2\varpi + 3V - 5E$	Annual increment	$= +36^{\circ}\cdot92$
$2\varpi + 2E - 4M$	,, ,,	$= +35\cdot73$
$\beta - 20V + 19E$	,, ,,	$= -31\cdot68$

As the annual increment of the argument of these terms lies between that of the two terms

J	Annual increment	$= +30^{\circ}\cdot67$

and

$\omega - \omega'$	,, ,,	$= +40\cdot67$

the existence of these disturbing terms should give rise likewise to apparent discordances in the values deduced from observation for the coefficients of the latter argument. References to Mr. Cowell's results show that this is the case, as he obtains the values

Period (i)	$+1''\cdot03 \sin(\omega-\omega') - 0''\cdot15 \cos(\omega-\omega')$
(ii)	$+ \cdot73 \sin(\ ,,\ ) - \cdot43 \cos(\ ,,\ )$
(iii)	$+ \cdot55 \sin(\ ,,\ ) - \cdot36 \cos(\ ,,\ )$

Mr. Cowell, however, as usual, ignores these large residuals depending on the cosine of the argument, but treats the coefficient of the sine of the argument as indicating that Hansen's tabular value for this term should be reduced from

$$+1''\cdot58 \sin(\omega-\omega')$$

to

$$+1\cdot00 \sin(\omega-\omega')$$

and therefore that Delaunay's value of

$$+0''\cdot87 \sin(\omega-\omega')$$

is more accurate than the value

$$+1''\cdot33 \sin(\omega-\omega')$$

deduced by Hansen in the *Darlegung*.

In the face of the large residuals depending on the cosine of this argument, the value deduced from the observations for the coefficient of the sine of the argument must be held to be uncertain by at least $\pm 0''\cdot40$, and hence no such conclusion

can be legitimately drawn. In fact, the true coefficient of this term lies between 1″·10 and 1″·18, being probably very close to +1″·13.

It will be noticed that the discordance in the coefficient of this term is much greater than even in the case of the term depending on J. This is because there are yet other terms exerting a systematic influence on these coefficients in addition to those already given; but lying on the other side of the argument $(\omega-\omega')$ they do not exert a corresponding influence on the coefficients of the J term. These are the terms depending on the arguments

$$\begin{aligned} g-21\mathrm{V}+21\mathrm{E} &= +43^\circ\cdot11 \quad \text{Annual increment} \\ 3\mathrm{V}-5\mathrm{E} &= -44\cdot53 \quad \text{,,} \quad \text{,,} \\ 2\mathrm{E}-4\mathrm{M} &= -45\cdot64 \quad \text{,,} \quad \text{,,} \end{aligned}$$

besides the term of different character depending on the argument

$$2\,\mathfrak{S} = -38^\circ\cdot68$$

In consequence of the existence of these disturbing terms the coefficients of this argument $(\omega-\omega')$ deduced by Mr. Cowell from the observations are really compound functions of the coefficients of at least seven other terms of similar period, and cannot be treated as giving any information as they stand as to the true correction to Hansen's tabular value of this term depending on the argument $(\omega-\omega')$.

Exactly the same thing occurs with the coefficients of the term to which M. Radau assigns the value

$$-0''\cdot112\,\sin(2\mathrm{E}-4\mathrm{M}+293^\circ)$$

Mr. Cowell deduces from observation the values

$$\begin{aligned} \text{Period (i)} \quad & -0''\cdot12\,\sin(2\mathrm{E}-4\mathrm{M}+82^\circ)-0''\cdot33\,\cos(2\mathrm{E}-4\mathrm{M}+82^\circ) \\ \text{(ii)} \quad & -\cdot13\,\sin(\quad\text{,,}\quad)+\cdot18\,\cos(\quad\text{,,}\quad) \\ \text{(iii)} \quad & +\cdot01\,\sin(\quad\text{,,}\quad)+\cdot11\,\cos(\quad\text{,,}\quad) \end{aligned}$$

Here, again, the residuals are far greater than can be disregarded, showing the values obtained to represent the combined effects of two or more similar terms. As the two terms depending on the arguments

$$2\mathrm{E}-4\mathrm{M}$$

and

$$3\mathrm{V}-5\mathrm{E}$$

only differ in annual increment by a little over a degree a year, the values deduced by Mr. Cowell must represent the combined effects of the two terms. The values given by Mr. Cowell do

not suffice to even roughly separate their values, but would indicate the values

$$+0''.20 \sin \{2E-4M+220°\}$$
$$+0''.30 \sin \{3V-5E+\ 70°\}$$

The values deduced from the observation are also affected by the terms depending on the arguments

$$g-(21V-21E)$$
$$\varpi-\varpi'$$

An important case is that of the term with the argument

$$E-J$$

Mr. Cowell deduces for the coefficient of this term the values

Periods (i) and (ii)	$+0''.68 \sin (E-J)$	$+0''.07 \cos (E-J)$
(iii)	$+\ .73 \sin (E-J)$	$+\ .13 \cos (E-J)$

In both cases the coefficient of the cosine term is too large to be disregarded as done by Mr. Cowell, and their existence shows that the values obtained are affected by some small allied term of closely similar period, and in consequence the coefficient deduced by Mr. Cowell must be regarded as uncertain by fully $\pm 0''.10$.

Large residuals of similar nature are also ignored by Mr. Cowell in connection with the arguments

$$(2\omega-2\omega') \qquad 2\omega \qquad g+2\omega \qquad 2g$$

and consequently in all these cases the corrections he has adopted to the tabular coefficients must be regarded as uncertain.

In a certain number of cases Mr. Cowell points out that the correction to the coefficients he deduces to two or more terms are so related that they cannot be regarded as independent; in fact, in these cases he takes into account the very conditions which he has neglected in the cases dealt with above. But he treats these cases somewhat arbitrarily, apparently swayed by the desire to bring the results as close to M. Radau's values as can be done, instead of deducing the values indicated by the results themselves by treating them as a system of simultaneous equations.

Thus dealing with the coefficients of the terms depending on the two arguments

$$D+g'$$
$$2D-g+3V-3E$$

Mr. Cowell points out that these two arguments differ so slightly

in period that for long intervals of time the coefficients deduced from the observations for the one term must involve the coefficients of the other. He obtains for the terms the following values :—

Period.	$\sin(D+g')$.	$\cos(D+g')$.	$\sin(2D-g+3V-3E)$.	$\cos(2D-g+3V-3E)$.
(i)	$+17''\cdot63$	$+0''\cdot04$	$-0''\cdot58$	$-0''\cdot02$
(ii)	$+17''\cdot01$	$-\ ''\cdot02$	$-\ ''\cdot90$	$+\ ''\cdot30$
(iii)	$+17''\cdot47$	$-\ ''\cdot29$	$-1''\cdot07$	$+\ ''\cdot27$

But instead of deducing from these values treated as simultaneous equations the actual results indicated by themselves, Mr. Cowell arbitrarily assumes the values

$$+17''\cdot50\sin(D+g')$$
$$-\ 0''\cdot70\sin(2D-g+3V-3E)$$

and points out that these values reduce the residuals left outstanding to the small quantities.

Period.	$\sin(D+g')$.	$\cos(D+g')$.	$\sin(2D-g+3V-3E)$.	$\cos(2D-g+3V-3E)$.
(i)	$-''\cdot24$	$+''\cdot15$	$-''\cdot09$	$-''\cdot08$
(ii)	$-\ ''\cdot07$	$-\ ''\cdot23$	$+\ ''\cdot03$	$+\ ''\cdot42$
(iii)	$-\ ''\cdot00$	$-\ ''\cdot29$	$+\ ''\cdot02$	$+\ ''\cdot16$

But here, again, the outstanding residuals are far larger than can be explained otherwise than by the existence of an associated term which Mr. Cowell has omitted to take into consideration.*

* The large value found from the observations of the period 1847–1901 for the cosine of the argument

$$D+g'=g+\omega-\omega'=g+\varpi-\varpi'$$

had specially attracted Mr. Cowell's attention in his earlier investigation (*Monthly Notices*, 1904 March, pp. 419–420). He then assigned to this term the value

$$-0''\cdot46\cos(g+\varpi-\varpi')$$

and suggested that its origin might be an error in the value, derived from theory, which he had adopted for the coefficient of the term depending on the argument

$$2D-g+3V-3E$$

or possibly that the true argument of the term might really be $g+\varpi$, the Moon's longitude ("since ϖ' is practically a constant, analyses of the errors cannot possibly distinguish between $g+\varpi$ and $g+\varpi-\varpi'$") and so may be due to systematic errors of observation. Later (*Monthly Notices*, 1904 June, p. 292) Mr. Cowell elaborates the idea and suggests that the principal portion of this term is really of the form

$$-0''\cdot40\cos(g+\varpi=l)$$

But this arbitrary treatment should be avoided and the values deduced from observation treated as a set of twelve simultaneous equations between the coefficients of the terms

$$\begin{aligned} &Q' \sin (D+g') + Q'' \cos (D+g') + V' \sin (2D-g+3V-3E) \\ &\qquad + V'' \cos (2D-g+3V-3E) \end{aligned}$$

and the values deduced from the normal equations

$$\begin{aligned} +4{\cdot}01Q' + {\cdot}00Q'' - {\cdot}80V' + {\cdot}02V'' &= +1''{\cdot}14 \\ + {\cdot}00 + 4{\cdot}01 - {\cdot}02 - {\cdot}80 &= + {\cdot}23 \\ - {\cdot}80 - {\cdot}02 + 4{\cdot}01 + {\cdot}00 &= +2{\cdot}33 \\ + {\cdot}02 - {\cdot}80 + {\cdot}00 + 4{\cdot}01 &= - {\cdot}90 \end{aligned}$$

which yield the values

$$\begin{aligned} V' &= +0''{\cdot}665 & Q' &= +0''{\cdot}414 \\ V'' &= -0{\cdot}247 & Q'' &= +0{\cdot}012 \end{aligned}$$

corresponding to the values

$$\begin{aligned} &+17''{\cdot}482 \sin (D+g') \\ &+ 0{\cdot}710 \sin (2D-g+3V-3E+340^\circ) \end{aligned}$$

and arises partly from the constant correction of $-1''{\cdot}00$ that Hansen has applied to all the tabular latitudes together with a possible constant error of similar amount in the observed declinations, due to refraction or some such cause, whilst the remainder might be due to an erroneous value of the lunar parallax. This latter view is repeated in the conclusion of his last paper (*Monthly Notices*, 1905 January, p. 275).

But Mr. Cowell has overlooked the fact that ϖ' though a constant has yet the value $279^\circ{\cdot}5$, so that

$$\begin{aligned} -0''{\cdot}46 \cos {}'g+\varpi-\varpi') = -0''{\cdot}46 \cos (g+\varpi-279^\circ{\cdot}5) &= -0''{\cdot}45 \sin (g+\varpi) \\ &\quad + 0''{\cdot}07 \cos (g+\varpi) \end{aligned}$$

so that any correction of the form

$$-0''{\cdot}40 \cos (g+\varpi=l)$$

would correspond with

$$-0''{\cdot}39 \sin (g+\varpi-\varpi')$$

and not with

$$-0''{\cdot}39 \cos (g+\varpi-\varpi')$$

Hence the cause he suggests will not explain the origin of this cosine term, though it does serve to explain the origin of a considerable part of the corresponding sine term.

In Table X. (*Monthly Notices*, 1904 December) Mr. Cowell has reduced the value of this cosine term to

$$-0''{\cdot}29 \cos (g+\varpi-\varpi')$$

but he gives no reason for the reduction.

The residuals left outstanding by this solution are

Period.	$\sin(D+g')$.	$\cos(D+g')$.	$\sin(2D-g+3V-3E)$.	$\cos(2D-g+3V-3E)$.
(i)	$-0''.22$	$+0''.06$	$-0''.12$	$-0''.20$
(ii)	$+ .00$	$+ .04$	$+ .02$	$- .08$
(iii)	$+ .14$	$- .16$	$+ .03$	$+ .14$

It will be seen that in this instance the values obtained for the coefficients of these terms does not differ from that assumed by Mr. Cowell by inspection, but that there is a sensible correction deduced to the epoch of the *Venus* term. The residuals are smaller than those left by Mr. Cowell's solution, and the large term depending on the cosine of the argument $(D+g')$ has been very much reduced. Yet the residuals which still exist are larger than they should be, and their gradual variation affords a strong indication that there exists an associated term of very similar period giving rise to periodical discordances of about three hundred years' period.

Such a term does exist, and depends on the argument

$$2D-g+4E-4M$$

The data given by Mr. Cowell do not serve to determine the value of the coefficient of this term, but it can easily be shown that the values

$$+0''.89 \sin(2D-g+3V-3E+340^\circ)$$
$$+0''.22 \sin(2D-g+4E-4M)$$

will represent the observed values with considerable accuracy. Both these terms have coefficients about a fifth of a second larger than that deduced from theory by M. Radau—a feature common to most of the coefficients of the planetary terms of this character when deduced from the observations.

Mr. Cowell adopts a similar method for dealing with the associated terms depending on the arguments

$$2D-g+2E-2J$$
$$g-\Omega$$

From the observations he deduces the values

Period.	$\sin(2D-g+2E-2J)$.	$\cos(2D-g+2E-2J)$.	$\sin(g-\Omega)$.	$\cos(g-\Omega)$.
(i)	$-1''.02$	$-0''.46$	$-0''.51$	$-0''.09$
(ii)	-1.44	$- .06$	$- .57$	$- .15$
(iii)	-1.13	$- .03$	$- .66$	$- .19$

He then assumes that the values of the coefficients are

$$-1''.00 \sin(2D-g+2E-2J) \qquad -0''.50 \sin(g-\Omega)$$

and calculates the consequent correction to the apparent values of the coefficients deduced from the observations showing that the adopted values leave outstanding the residuals

Period.	$\sin(2D-g+2E-2J)$.	$\cos(2D-g+2E-2J)$.	$\sin(g-\Omega)$.	$\cos(g-\Omega)$.
(i)	+0".17	+0".01	+0".03	−0".19
(ii)	+ .06	− .08	+ .04	− .15
(iii)	− .12	− .05	− .06	− .10

These are too large, especially in the case of the cosine of the argument $(g-\Omega)$.

Treating the values as twelve simultaneous equations in the same manner as before, the resulting normal equations yield the values

$$-1".015 \sin(2D-g+2E-2J+355°)$$
$$-0".525 \sin(g-\Omega)$$
$$-0".181 \cos(g-\Omega)$$

leaving residuals in all cases under $\pm 0".05$.

The data given by Mr. Cowell in an earlier paper (*Monthly Notices*, 1904 March, p. 417) are more complete, and suffice to determine the coefficient of the allied term depending on the argument $(g-2\varpi+2J)$, as well as that of the *Jovian evection*. From the observations for the period 1847–1901 (iii) he deduces the values

$$\{-0".02 \sin(2\varpi-2J)-0".98 \cos(2\varpi-2J)\} \sin g$$
$$\{-1".26 \sin(2\varpi-2J)+0".17 \cos(2\varpi-2J)\} \cos g$$

from which, by omitting small terms, he derives the value

$$-1".12 \sin(g+2\varpi-2J)$$

The complete values are

$$-1".123 \sin(g+2\omega-2J)+0".075 \cos(g+2\varpi-2J) = -1".125 \sin(g+2\varpi-2J+356°)$$
$$+0".140 \sin(g-2\omega+2J)+0".095 \cos(g-2\varpi-2J) = +0".169 \sin(g-2\varpi+2J+34°)$$

It will be seen that the small terms neglected by Mr. Cowell are nearly as large as the correction he adopts and larger than many terms he takes into account.

At the same time Mr. Cowell gives the results

$$+0".52 \cos(2\varpi-3J+7°) \sin g+0".21 \sin(2\varpi-3J+7°) \cos g$$

from which he deduces the value

$$+0".36 \sin(g+2\varpi-3J+7°)$$

but ignores the corresponding value

$$+0''\cdot16\sin(g-2\varpi+3\mathrm{J}-7^\circ)$$

Yet this second term must be held as well established as the first, for if ascribed to accidental errors it must render the former too uncertain to possess any value. In his final values the results for this period are changed to

$$+0''\cdot42\sin(g+2\varpi-3\mathrm{J}+7^\circ)+0''\cdot13\cos(g+2\varpi-3\mathrm{J}+7^\circ)$$

and Mr. Cowell ignores the cosine term as usual.

The pair of terms depending on the arguments

$$3\mathrm{V}-4\mathrm{E}-2^\circ=\mathrm{E}+(3\mathrm{V}-5\mathrm{E})-2^\circ$$

and

$$g'-\omega+\omega'=g'-(\varpi-\varpi')$$

though bracketed by Mr. Cowell, on p. 135 of Table X., as requiring simultaneous treatment, have not been referred to in the notes, where the remainder of the associated terms are examined and their corrected values deduced. The values deduced from the observations are :—

	$\sin(3\mathrm{V}-4\mathrm{E}-2^\circ)$.	$\cos(3\mathrm{V}-4\mathrm{E}-2^\circ)$.	$\sin(g'-\varpi+\varpi')$.	$\cos(g'-\varpi+\varpi')$.
Periods (i) and (ii)	$-0''\cdot16$	$-0''\cdot27$	$+18''\cdot43$	$+''\cdot23$
(iii)	$-\cdot10$	$-\cdot08$	$+18\cdot68$	$-\cdot32$

and Mr. Cowell adopts the values

$$-0''\cdot10\sin(3\mathrm{V}-4\mathrm{E}-2^\circ)\qquad+18''\cdot60\sin(g'-\varpi+\varpi')$$

ignoring, as usual, the large cosine terms.

Yet with more care a much better result can be obtained from Mr. Cowell's data. For the united periods (i) and (ii) the two terms are independent of each other, and the values found may be taken as they stand as the values of their respective coefficients. The period (iii) extends over too short a period for this to be done, and the true values must be deduced from the simultaneous equations. They are

Period (iii)	...	$-0''\cdot44$	$-0''\cdot03$	$+18''\cdot84$	$-0''\cdot53$

These correspond to the terms

Periods (i) and (ii)	$+0''\cdot31\sin(3\mathrm{V}-4\mathrm{E}+237^\circ)$	$+0''\cdot28\sin(g-\varpi+\varpi'+126^\circ)$
(iii)	$+0\cdot44\sin(3\mathrm{V}-4\mathrm{E}+182)$	$+0\cdot58\sin(g-\varpi+\varpi'+294)$

in addition to the theoretical term

$$+18''\cdot60\sin(g-\varpi+\varpi')$$

To bring the constant factors of the preceding terms into harmony the argument of the *Venus* term must have its annual increment decreased by $55° \div 73 = 0°{\cdot}76$, and the annual increment of the second term must be increased by $168° \div 73 = +2°{\cdot}31$, making the annual increment of the two arguments differ from that of the Earth's mean longitude by $-45°{\cdot}20$ in the first case and by $-38°{\cdot}36$ in the second case. At the same time the decrease in the coefficients of the two terms shows that in each case it must represent the sum of at least two terms. In the first case the values deduced from the observations is the united effect of the terms depending on the arguments

$$3V-4E$$

and

$$3E-4M$$

Mr. Cowell's data do not suffice to determine the separate values of the coefficients of these two terms, but it can easily be shown that the observed values are entirely satisfied by the values

$$+0''{\cdot}548 \sin (3V-4E+201^\circ)$$
$$+0{\cdot}284 \sin (3E-4M+39)$$

These terms illustrate how two terms of considerable magnitude may for long periods mutually counterbalance each other and leave outstanding a compound term of much less magnitude. In the case of the second term the observed discordances can be explained by the allied terms depending on the pair of arguments

$$2D-2g+3V-4E$$
$$2D-2g+3E-4M$$

The most important instances of closely associated terms are those depending on the arguments

$$2M-E+49^\circ$$
$$2\varpi-2J$$
$$-\Omega$$

For the apparent value of the coefficients of these arguments Mr. Cowell obtains from the observations

	Period (i). Value.	Residual.	Period (ii). Value.	Residual.	Period (iii). Value.	Residual.	
$\sin(2M-E+49^\circ)$	$-''{\cdot}67$	$-''{\cdot}12$	$-''{\cdot}52$	$+''{\cdot}02$	$+''{\cdot}09$	$+''{\cdot}03$	$=M'$
$\cos(2M-E+49)$	$-{\cdot}50$	$+{\cdot}06$	$+{\cdot}50$	$-{\cdot}02$	$+{\cdot}12$	$-{\cdot}17$	$=M''$
$\sin(2\varpi-2J)$	$+{\cdot}67$	$-{\cdot}05$	$+{\cdot}39$	$-{\cdot}19$	$+{\cdot}59$	$+{\cdot}05$	$=J'$
$\cos(2\varpi-2J)$	$-{\cdot}87$	$+{\cdot}31$	$-{\cdot}62$	$+{\cdot}04$	$-{\cdot}73$	$-{\cdot}23$	$=J''$
$\sin(-\Omega)$	$-6{\cdot}95$	$-{\cdot}18$	$-6{\cdot}39$	$-{\cdot}28$	$-6{\cdot}15$	$+{\cdot}40$	$=B'-6''{\cdot}60$
$\cos(-\Omega)$	$-{\cdot}97$	$-{\cdot}06$	$-{\cdot}66$	$+{\cdot}03$	$-{\cdot}59$	$-{\cdot}00$	$=B''$

To represent these values Mr. Cowell assumes that the true coefficients of the terms are

$$-0''.40 \sin(2M-E+49^\circ)$$
$$+0''.20 \sin(2\varpi-2J)$$
$$-6''.60 \sin(-\Omega)$$
$$-0''.70 \cos(-\Omega)$$

The outstanding residuals left by the substitution of these values is given under the heading "Residuals." They are considerable in magnitude, nearly half of them considerably exceeding a tenth of a second of arc, and the solution fails to represent even the latest and most accurate values of period (iii). In adopting these values Mr. Cowell appears to have been anxious to reduce the planetary terms to the small values indicated by M. Radau's theoretical calculations, and in so doing has thrown over the much more accurately calculated values of Mr. Hill for the terms depending on the figure of the Earth. But unless there is some serious defect in the theoretical basis of Mr. Hill's calculation of the values of the terms, due to the figure of the Earth, the values he has deduced must be close approximations to the true values, and cannot admit of such corrections as Mr. Cowell indicates. Is there any reason for supposing the method adopted to be defective? Hansen has obtained similar results by an entirely different method of admitted adequacy. Yet, unless the existence of these large errors be admitted, it follows that to represent the observations very much larger values must be adopted for the coefficients of other terms, due to the disturbing action of the planets; and it is solely by thus throwing the whole error on the theoretical coefficients of the terms depending on the longitude of the lunar node that it is possible to reduce the coefficients of the planetary terms to the small dimensions that have been adopted by Mr. Cowell.

The only satisfactory manner of treating this question is to derive the values directly from the observed results by solving the system of eighteen simultaneous equations by the method of least squares. Adopting Mr. Cowell's data, there are obtained the system of normal equations

$+6''.31M' +$	$''.00M'' -$	$1''.06J' -$	$''.31J'' +$	$1''.31B' -$	$''.35B'' =$	$-2''.35$
$+''.00$	$+6''.31$	$+''.31$	$-1''.06$	$+''.35$	$+1''.31$	$=-1''.08$
$-1''.06$	$+''.31$	$+7''.69$	$+''.00$	$+4''.70$	$-''.31$	$=+1''.79$
$-''.31$	$-1''.06$	$+''.00$	$+7''.69$	$+''.31$	$+4''.70$	$=-4''.39$
$+1''.31$	$+''.35$	$+4''.70$	$+''.37$	$+6''.85$	$-''.00$	$=-''.10$
$-''.35$	$+1''.31$	$-''.31$	$+4''.70$	$+''.00$	$+6''.85$	$=-5''.41$

yielding the values

$$M' = -0''.372 \qquad J' = +0''.198 \qquad B' = -0''.066$$
$$M'' = -0.074 \qquad J' = -0.196 \qquad B'' = -0.652$$

corresponding to the term

$$-0''.379 \sin(2M - E + 60°)$$
$$+0.279 \sin(2\omega - 2J + 315°)$$
$$-6.666 \sin(-\Omega)$$
$$-0.652 \cos(-\Omega)$$

It will be seen that these results confirm those adopted by Mr. Cowell in throwing the greater portion of the correction on to the coefficients of the theoretical ellipticity terms, yielding the above values instead of the theoretical values

$$-7''.708 \sin(-\Omega)$$
$$-0.292 \cos(-\Omega)$$

The residuals left by these new values are smaller than those left by Mr. Cowell's solution, but the observed and calculated values of the coefficients of the Moon's node still show the large residuals.

	Period (i).	Period (ii).	Period (iii).
$\sin(-\Omega)$	$-''.02$	$-''.17$	$+''.33$
$\cos(-\Omega)$	$+.00$	$+.13$	$+.10$

Some disturbing cause must still remain outstanding. The residuals could be represented by increasing the coefficient of the term depending on the argument $(2\varpi - 2J)$ by one half, and introducing a small term depending on an argument of the form

$$\varpi + E - 2M$$

but the data given by Mr. Cowell are not sufficient to justify this course being adopted.*

* In the preceding I have assumed that Mr. Cowell has correctly stated the modified tabular coefficient used in Period (iii) as

$$-7''.34 \sin(-\Omega) - 0''.71 \cos(-\Omega)$$

But I can find no reference in his papers to the application of any correction to Hansen's tabular coefficient of *sin* (Ω); and if no correction has been applied the value used must be Hansen's tabular value unaltered or

$$-8''.12 \sin(-\Omega)$$

If by an oversight Mr. Cowell had subtracted instead of added the portion of the complete coefficient of the term in the expression for the true longitude

The results obtained indicate very strongly that the action of the planets gives rise to perturbations which are explicitly independent of the Moon's longitude, having coefficients much larger than those which can be theoretically deduced from the combined first powers of the disturbing action of the Sun and planets.

(6) It remains now to compare the values of the terms of longer period which have been deduced by Mr. Cowell from the observations, with the standard observed errors of the tables.

Let "Hansen (modified)" denote Hansen's tabular value after the removal of all the planetary terms. Then the correction to this modified value deduced by Mr. Cowell from the discussion of the observations, after omitting the smaller correction to the terms of shorter period, may be written as

which is derived from the reduction to the ecliptic, it would account for the discrepancy as

$$-\{+7''\cdot734-0''\cdot388=+7''\cdot346\}\sin(-\Omega)$$

instead of

$$-\{+8''\cdot122\}\sin(-\Omega)$$

But in this case the preceding would want revising, as the observed value for the Period (iii) would be

$$-6''\cdot94\sin(-\Omega)$$

instead of

$$-6''\cdot15\sin(-\Omega)$$

and Mr. Cowell's residual be $-0''\cdot38$ instead of $+0''\cdot40$.

The amended normal equation yield now the values

$$-0''\cdot556\sin(2M-E+57^\circ)$$
$$+0\cdot173\sin(2\omega-2J+1^\circ)$$
$$-6\cdot872\sin(-\Omega)$$
$$-0\cdot794\cos(-\Omega)$$

and the large residuals in sin Ω are much decreased, they being

	Period (i).	Period (ii)	Period (iii).
$\sin(-\Omega)$	$+\cdot18$	$-\cdot08$	$+\cdot01$
$\cos(-\Omega)$	$+\cdot07$	$+\cdot06$	$+\cdot13$

If this view be correct, then the residuals between Mr. Cowell's adopted and the observed values of the coefficient of sin Ω would be

Period (i) $=-0''\cdot18$ Period (ii) $=-0''\cdot28$ Period (iii) $=-0''\cdot38$

so that he would have increased his adopted value by about $-0''\cdot30$ or brought it up to

$$-6''\cdot90\sin(-\Omega)$$

leaving the residuals

Period (i) $=+0''\cdot12$ Period (ii) $=+0''\cdot02$ Period (iii) $=-0''\cdot08$

Cowell's Correction $=$

$$
\begin{aligned}
&-\ 2''.84\\
&-30.67(T-180^\circ)\\
&-\ 5.96(T-180^\circ)^2\\
&+14.40 \sin \{18V-16E-g+30.6\}\\
&+\ 0.27 \sin \{\ 8V-13E+228.0\ \}\\
&-\ 0.80 \sin \{\ V-\ E\ \}\\
&+\ 0.30 \sin \{\ 2V-\ 2E\ \}\\
&-\ 0.30 \sin \{\ 2V-\ 3E+85\ \}\\
&-\ 0.10 \sin \{\ 3V-\ 4E+88\ \}\\
&+\ 0.40 \sin \{\ E-\ 2M-49\ \}\\
&+\ 0.30 \sin \{\ 2E-\ 2M\ \}\\
&+\ 0.15 \sin \{\ 2E-\ 4M+317\ \}\\
&+\ 0.70 \sin \{\ E-\ J\ \}\\
&+\ 0.20 \sin \{\ E-\ 2J+298\ \}\\
&-\ 0.20 \sin \{\ 2E-\ 2J\ \}\\
&-\ 0.10 \sin \{\ J+99\ \}\\
&+\ 0.20 \sin \{\ 2D-2g+(2E-2J)\}\\
&-\ 0.10 \sin \{\ g-(3V-3E)\ \}\\
&-\ 0.30 \sin \{\ g-(2E-2J)\ \}\\
&+\ 0.10 \sin \{\ 2D-g-(V\ -\ E)\}\\
&-\ 0.10 \sin \{\ 2D-g-(2V-2E)\}\\
&-\ 0.70 \sin \{\ 2D-g+(3V-3E)\}\\
&+\ 0.20 \sin \{\ 2D-g+(E\ -\ J)\}\\
&-\ 0.20 \sin \{\ 2D-g-(E-J)\ \}\\
&-\ 1.10 \sin \{\ 2D-g+(2E-2J)\}\\
&+\ 0.40 \sin \{\ 2D-g+(2E-3J)\}\\
&-\ 0.10 \sin \{\ 2D-(V-E)\ \}\\
&-\ 0.10 \sin \{\ 2D-(2V-2E)\ \}\\
&-\ 0.20 \sin \{\ 2D-(E-J)\ \}\\
&-\ 1.22 \sin \{\ \Omega\ \}\\
&+\ 0.23 \sin \{\ g+\Omega\ \}\\
&-\ 0.23 \sin \{\ g-\Omega\ \}\\
&-\ 0.58 \sin \{\ \varpi-\varpi'\ \}\\
&+\ 0.07 \sin \{\ 2\varpi-2\varpi'\ \}\\
&+\ 0.15 \sin \{\ 2\varpi\ \}\\
&-\ 0.55 \sin \{\ g'\ \}\\
&-\ 0.55 \sin \{\ g'-\varpi+\varpi'\ \}
\end{aligned}
$$

$$\begin{aligned}
\text{Cowell's corrections} = &- 0''.67 \sin \{ g \} \\
&+ 1''.56 \sin \{ D \} \\
&- 0''.67 \sin \{ D+g' \} \\
&- 0''.25 \sin \{ 2D-g \} \\
&+ 12''.16 \sin \{ 1''.10[Y-1850.5]+153^\circ \} \\
&+ 2''.35 \sin \{ 5.43[Y-1850.5]+ 55 \} \\
&+ 0''.73 \sin \{ 8.60[Y-1850.5]+ 15 \} \\
&- 1''.50
\end{aligned}$$

The three last inequalities are Mr. Cowell's three empirical terms, and to them has been added the further correction to the epochs which has been already shown to be indicated by the observations.

Then the comparison between the calculated and observed mean yearly error of Hansen's tables yields the values given beneath under the heading "Cowell" for the discordance between the amended tables and the Greenwich Transit Circle observations :—

	"Cowell."		"Cowell Supplemented."			"Cowell."		"Cowell Supplemented."	
	Annual.	Tri-annual.	Annual.	Tri-annual		Annual.	Tri-annual.	Annual.	Tri-annual.
1902	−1".03	...	−".47	...	1883	−1".27	−1".14	−".29	−".16
01	−.62	−1.01	−.21	−.54	82	−1.17	...	−.18	...
1900	−1.39	...	−.94	...	81	−1.58	...	−.58	...
1899	−.15	...	+.35	...	80	−1.60	−1.49	−.60	−.49
98	−.66	−.61	−.12	−.07	79	−1.29	...	−.29	...
97	−1.02	...	−.43	...	78	−1.25	...	−.25	...
96	−.56	...	+.07	...	77	−.53	−.48	+.47	+.52
95	−.75	−.75	−.08	−.08	76	+.34	...	+1.35	...
94	−.95	...	−.24	..	75	−.29	...	+.67	...
93	−.94	...	−.20	...	74	−.43	−.23	+.52	+.72
92	−1.35	−1.31	−.57	−.54	73	+.05	...	+.98	...
91	−1.65	...	−.84	...	72	−.51	...	+.40	...
90	−1.99	...	−1.15	...	71	−.07	−.38	+.82	+.51
89	−1.33	−1.44	−.46	−.68	70	−.56	...	+.31	...
88	−1.33	...	−.44	...	69	−.33	...	+.51	...
87	−.95	...	−.04	...	68	+.21	−.04	+1.02	+.77
86	−.14	−.65	+.79	+.28	67	+.01	...	+.78	...
85	−.85	...	+.10	...	66	+.09	...	+.83	...
84	−.97	...	+.00	...	65	−.42	−.35	+.29	+.36

	"Cowell."		"Cowell Supplemented."			"Cowell."		"Cowell Supplemented."	
	Annual.	Tri-annual.	Annual.	Tri-annual.		Annual.	Tri-annual.	Annual.	Tri-annual.
	″	″	″	″		″	″	″	″
1864	−·71	...	−·04	...	1857	−1·43	...	−1·07	...
63	−1·12	...	−·49	...	56	−1·45	−1·31	−1·14	−1·00
62	−1·12	−1·16	−·53	−·57	55	−1·04	...	−·78	...
61	−1·25	...	−·70	...	54	−1·12	...	−·91	...
60	−1·30	...	−·80	...	53	−·81	−1·04	−·65	−·88
59	−1·20	−1·27	−·75	−·81	52	−1·20	...	−1·09	...
58	−1·30	...	−·89	...	1851	−·77	...	−·72	...

It is apparent that a further correction to the epoch of −0″·80 would bring the calculated and observed values into much closer accord, but such a correction cannot be made without throwing the earlier observations, made between 1850 and 1750, into discordance with observation far greater than is permissible ; for if the observations from 1900–1850 are such as to require a decrease of the epoch by about 0″·8, the observations for 1820–1770 require an increase of at least as much.

The comparison indicates a term of approximately the form

$$-1''\cdot00 \sin \{3°\cdot0[Y-1850\cdot5]\}$$

Supposing Mr. Cowell's values supplemented by the addition of such a term, the discordances are reduced to the values shown under the heading "Cowell Supplemented." It brings the calculated and the observed errors into much closer agreement. Probably a still better result, as far as these modern observations are concerned, would be gained by using a correction of the form

$$-0''\cdot50-0''\cdot50 \sin \{2°\cdot8[Y-1850\cdot5]\}$$

It will be seen that Mr. Cowell's corrections, when thus supplemented, serve to reduce the tri-annual mean discordances between computation and observation to quantities considerably under a second of arc, which is a great step in advance, though it does not reach the degree of accuracy which should be attained by a satisfactory solution of reducing the discordance to under ±0″·50. The annual means are more discordant, but they are liable to sudden violent fluctuation, due mainly to changes of observers, but partly to meteorological conditions, so that, unless steps are taken to eliminate the effects of these, sudden changes of ±0″·50 may occur in the mean for the year.

A critical examination of the run of the discordances shows clear indication that there yet remain outstanding sensible inequalities of long period which have been omitted by Mr. Cowell. One of about thirty-six years and two of about nine years seem clearly indicated. Increasing all Mr. Cowell's values for the

coefficients of the planetary terms by about a fourth would also sensibly decrease the discordance between computation and observation.

(7) The consideration of the correction of Class III. (§ 3) is postponed for the present, pending further leisure. The value deduced for the "parallactic inequality" requires considerable correction, for the effect of systematic errors he has not taken into account.

(8) In conclusion I must congratulate Mr. Cowell on the degree of success which he has attained in dealing with the long series of Greenwich observations, and especially in showing so clearly that the theoretical values for the perturbations of the Moon by the Sun are in entire harmony with the results of observation. I can appreciate it all the more, as for years I had worked on the same subject; but my results, more detailed and elaborated than Mr. Cowell's, have for years been lying awaiting official publication, and will, I fear, have to wait for several years longer until the revenue of the colony is again normal; so Mr. Cowell's results take precedence, and mine must wait. I could have supplemented Mr. Cowell's data by furnishing the value derived from both observation and theory for most of the terms due to the planetary perturbations, but judged it preferable to keep entirely to the data furnished by Mr. Cowell's successful discussion of the observations.

Discussion of Greenwich Observations of the Sun, 1864–1900.
By P. H. Cowell, M.A.

An observation of a planet from the Earth being the exact counterpart of an observation of the Earth from the planet, it follows that the elements of the Earth's orbit can be obtained from observations of the planets. I hope to analyse the apparent errors of the Greenwich meridian observations of the planets for the years during which Le Verrier's tables were used in the *Nautical Almanac*, and I have already prepared such an analysis for the observations of *Venus*. The corrections to the elements of the Earth's orbit obtained directly from the Sun evidently form a useful check upon the results of the more complex discussion for a planet.

In this paper I only consider the right ascensions of the Sun, and I have only analysed for terms whose arguments are the longitude of the Earth and the difference of longitude of the Earth and *Venus*. In this way corrections to the solar eccentricity and perigee are obtained, and also a correction is obtained to the mass of *Venus*.

The figures to be analysed are given in Table I. The unit is $0^s.01$; the quantity tabulated is the mean error of right ascension for the month. Each month receives equal weight in the analysis, and the error for 1891 September has been supplied by interpolation.

TABLE I.

Mean Error of Sun's R.A. 1864–1900. *Unit* 0ˢ·01.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1864	− 2	− 4	− 4	+ 3	− 7	− 3	+ 4	+ 3	− 2	+ 3	+ 2	+ 7
1865	− 2	− 1	− 3	+ 3	+ 4	− 1	+ 2	+ 1	+ 10	+ 1	+ 4	+ 8
1866	− 4	0	− 4	− 1	− 7	− 1	0	+ 2	− 2	+ 5	+ 2	− 7
1867	− 8	−10	+ 1	− 5	− 3	− 7	+ 5	+ 2	0	0	0	+ 3
1868	− 6	− 5	− 6	− 3	− 3	+ 5	+ 2	+ 1	+ 10	+ 5	+ 9	− 3
1869	− 1	− 9	−13	0	+ 4	− 4	+ 6	+ 3	+ 6	+ 4	+ 3	+ 7
1870	− 3	− 5	− 3	− 2	− 1	− 1	+ 2	+ 1	+ 3	+ 6	+ 3	+ 11
1871	+ 2	0	− 3	− 3	+ 3	+ 3	+ 5	+ 1	+ 4	+ 1	+ 9	+ 3
1872	+ 2	− 3	− 9	− 5	− 4	0	+ 1	+ 1	+ 3	+ 4	+ 4	+ 2
1873	+ 2	+ 3	+ 3	− 3	+ 3	+ 5	− 1	+ 2	+ 7	+ 6	+ 6	+ 9
1874	+ 4	+ 4	+ 3	+ 1	+ 10	+ 6	+ 1	+ 6	+ 5	+ 8	+ 10	+ 9
1875	+ 8	+ 5	+ 3	− 3	+ 3	+ 10	+ 5	+ 2	+ 10	+ 6	+ 11	+ 4
1876	+ 2	+ 2	0	+ 2	+ 7	+ 7	+ 6	+ 7	+ 5	+ 3	+ 7	+ 4
1877	+ 5	0	− 2	− 6	+ 7	+ 5	+ 3	+ 4	− 1	+ 16	+ 8	+ 4
1878	+ 8	+ 2	+ 1	+ 1	− 2	+ 6	+ 9	+ 6	+ 5	+ 5	+ 12	+ 7
1879	+ 4	+ 3	+ 1	+ 4	− 1	+ 2	+ 6	+ 5	+ 3	+ 11	+ 10	+ 10
1880	+ 6	+ 4	− 5	0	+ 3	+ 11	+ 3	+ 6	+ 7	+ 5	+ 12	+ 19
1881	+ 6	+ 6	+ 3	0	+ 4	+ 3	+ 9	+ 5	+ 4	+ 7	+ 8	+ 4
1882	+ 7	+ 2	0	− 1	+ 1	+ 8	+ 2	+ 2	+ 3	+ 4	+ 8	+ 5
1883	+ 7	+ 4	+ 4	+ 2	+ 5	0	+ 6	+ 5	+ 4	+ 5	+ 6	+ 6
1884	+ 3	+ 1	+ 2	+ 2	+ 3	+ 12	+ 2	+ 8	+ 8	+ 6	+ 14	+ 12
1885	+ 9	− 4	+ 3	+ 5	+ 1	+ 7	+ 2	+ 1	+ 9	+ 8	+ 13	+ 7
1886	+ 10	+ 3	+ 2	+ 4	+ 4	+ 7	+ 8	+ 5	+ 9	+ 8	+ 12	+ 7
1887	+ 8	+ 5	+ 2	+ 4	+ 5	+ 4	+ 1	+ 9	+ 6	+ 5	+ 6	+ 8
1888	+ 5	+ 4	+ 6	0	+ 1	+ 3	+ 2	+ 2	+ 7	+ 4	+ 11	+ 7
1889	+ 7	+ 8	+ 5	+ 1	+ 3	+ 10	+ 4	+ 2	+ 9	+ 6	+ 13	+ 9
1890	+ 8	+ 5	+ 2	+ 4	− 3	0	+ 2	+ 8	+ 4	+ 4	+ 13	+ 11
1891	+ 7	+ 3	+ 6	+ 6	+ 5	+ 7	+ 8	+ 14	(+ 7)	+ 8	+ 14	+ 7
1892	+ 7	+ 3	0	+ 7	+ 6	+ 9	+ 10	+ 6	+ 7	+ 12	+ 16	+ 14
1893	+ 13	+ 8	+ 6	+10	+ 10	+ 7	+ 10	+ 9	+ 11	+ 7	+ 14	+ 5
1894	+ 12	+11	+ 8	+ 6	+ 4	+ 14	+ 13	+ 13	+ 17	+ 12	+ 16	+ 15
1895	+ 15	+11	+ 6	+ 7	+ 11	+ 10	+ 13	+ 10	+ 12	+ 12	+ 11	+ 12
1896	+ 10	+ 8	+ 6	+ 8	+ 8	+ 13	+ 9	+ 11	+ 10	+ 14	+ 16	+ 14
1897	+ 9	+ 8	+ 7	+ 4	+ 10	+ 10	+ 14	+ 14	+ 15	+ 17	+ 16	+ 10
1898	+ 12	+11	+ 5	+ 7	+ 11	+ 7	+ 8	+ 11	+ 16	+ 17	+ 14	+ 15
1899	+ 16	+ 5	+ 8	+ 8	+ 10	+ 9	+ 12	+ 13	+ 10	+ 17	+ 10	+ 17
1900	+ 11	+10	+ 9	+ 6	+ 6	+ 11	+ 8	+ 8	+ 11	+ 14	+ 16	+ 9
Sums	+199	+98	+50	+73	+121	+194	+202	+209	+252	+276	+359	+291

The sum of all the quantities in Table I. is +2324.

The tabular minus observed epoch is therefore $+23^{s}\cdot24 \div 444 = +0^{s}\cdot052 = +0''\cdot78$.

Analysing the sums for the different months and treating E, the Earth's longitude, as 115° for January, 145° for February, and so on, we get

$$(\text{tab.}-\text{obs.}) \quad +0''\cdot78-0''\cdot47\sin(E-115^\circ)+0''\cdot00\cos(E-115^\circ)$$
$$+(-0''\cdot21\sin+0''\cdot04\cos)2(E-115^\circ)$$

No appreciable part of the last term will correspond to possible errors of eccentricity or obliquity. The most probable explanation is instrumental error. The preceding term $-0''\cdot47\sin(E-115^\circ)$ indicates that Le Verrier's eccentricity requires an increase. Here, again, instrumental error probably affects the result. We know, for instance, that there is a diurnal change in the level error whose amplitude is greater in summer than in winter. The effect of this could be roughly estimated, but it seems preferable to analyse the errors as they stand and merely point out that the results are liable to small systematic errors. The extent of the observations (thirty-seven years) is too short to obtain reliable corrections proportional to the time.

The corrections to Le Verrier's longitude in 1882 is therefore

$$-0''\cdot78+0''\cdot47\sin(E-115^\circ)$$
$$= -0''\cdot78+0''\cdot46\sin(E-100^\circ)-0''\cdot12\cos(E-100^\circ)$$

Element.	Correction required by Le Verrier in 1882.
Mean longitude	$-0''\cdot78$
Eccentricity	$+0''\cdot23$
Coefficient of principal elliptic term in longitude	$+0''\cdot46$
Mean anomaly	$-0''\cdot12\div2e=-3''\cdot6$
Perihelion	$+2''\cdot8$

It is interesting to compare these corrections with the differences between Le Verrier's tables and Newcomb's.

Counting T in centuries from 1900·0 we have the following differences, Newcomb–Le Verrier.

Element.	*Formula.* For Differences Newcomb–Le Verrier.	Value in 1882.
Mean longitude ...	$-0''\cdot71-0''\cdot82T-0''\cdot0183T$	$-0''\cdot56$
Perihelion	$+7''\cdot8+17''\cdot26T-0''\cdot19T^2+0''\cdot012T^3$	$+4''\cdot7$
Principal elliptic coefficient in longitude	$+0''\cdot52+0''\cdot27T+0''\cdot004T^2$	$+0''\cdot47$

There is most satisfactory agreement between the eccentricity found in this paper and that used by Newcomb. The mean longitude and perihelion cannot be compared unless allowance is made

(i) For any changes of long-period terms between Le Verrier's tables and Newcomb's.

(ii) For the difference $0''{\cdot}8$ between Newcomb's equinox and that used in the Greenwich clock-star list up to and including 1902.

Coming now to the analysis with the heliocentric elongations of *Venus* as argument, an auxiliary angle with nineteen different values

$$180^\circ + \frac{p \times 360^\circ}{19}$$

was employed.

Approximately the value of p increases by unity in each month, that is to say, for each entry of Table I.; but in order to allow for the slight excess of the synodic period of *Venus* over nineteen Julian months, the value $p = 4$ is assigned to both 1868 January and February; $p = 14$ to both 1874 November and December; $p = 5$ to both 1885 September and October; $p = 15$ to 1894 July and August. We thus obtain

TABLE II.

V−E.	p.	Sum of Errors.	No.
198°·9	1	+106	23
217·9	2	+118	23
236·8	3	+104	23
255·8	4	+104	24
274·7	5	+138	24
293·7	6	+109	23
312·6	7	+121	23
331·6	8	+125	23
350·5	9	+130	23
9·5	10	+124	23
28·4	11	+143	23
47·4	12	+111	23
66·3	13	+127	24
85·3	14	+149	25
104·2	15	+128	25
123·2	16	+119	23
142·1	17	+ 97	23
161·1	18	+133	23
180·0	19	+138	23

Analysing in the usual way we have

$$(\text{tab.}-\text{obs.}) \quad +0''\cdot03 \sin (V-E) + 0''\cdot00 \sin 2(V-E)$$

Newcomb, using a mass of *Venus* 1 : 408000, obtains terms in the Earth's longitude

$$-4''\cdot838 \sin (V-E) + 5''\cdot526 \sin 2(V-E)$$

Le Verrier, using a mass of *Venus* 1 : 401847, in his theory obtains

$$-4''\cdot913 \sin (V-E) + 5''\cdot613 \sin 2(V-E)$$

but in his tables he uses a mass (*Annales de l'Obs. de Paris*, vol. iv. p. 34) 1 : 400246 (p. 102) with inequalities

$$-4''\cdot93 \sin (V-E) + 5''\cdot63 \sin 2(V-E)$$

To these terms the apparent corrections just obtained are

$$-0''\cdot03 \sin (V-E) + 0''\cdot00 \sin 2(V-E)$$

For the mass of *Venus* we therefore have

from the term in	V−E	1·006 to 400246
„	„ 2(V−E)	1·000 „ 400246
	Mean	1·003 to 400246.

Newcomb, in his provisional theory (*Astron. Constants*, pp. 7, 8), uses 1 : 401847, and on p. 22, last six lines, he obtains a correction +·007 from the Greenwich observations 1867–1892, in close accordance with the result of this paper. The present paper, however, supplements Newcomb's calculations by considering the two principal terms separately instead of all the terms together.

Professor Newcomb, however, swamps the result obtained from recent Greenwich observations by introducing additional material of decidedly inferior value, and on pp. 101–2 he reaches the result 1 − ·0119 : 401847, and in his tables of the Sun he uses 1 : 408000, as stated above.

This latter value would imply that the two coefficients as obtained from the thirty-seven years 1864 to 1900 are respectively 0″·12 and 0″·10 in error. I do not believe that such errors are possible in the case of terms of short periods different from the periods of possible instrumental errors ; certainly in my analyses of the errors of the Moon I found no such discordances in cases where comparison could be made with well-determined theoretical values.

Moreover, in the older observations I did find large discordances in the case of the Moon, and I can easily believe that they may also exist in the case of the Sun. It must be remembered that the older observations of the Sun and Moon have not been

re-reduced with the same care that has lately been bestowed on several of the earlier star catalogues. I consider therefore that Professor Newcomb has given excessive weight to his other material, and that it would have been better to leave Le Verrier's mass of *Venus* unaltered.

Reciprocal of Mass of *Venus*.

Le Verrier	400×10^3
Newcomb	408×10^3
Present paper	399×10^3

Discussion of Greenwich Observations of Venus, 1869–1900.
By P. H. Cowell, M.A.

The observations under examination are divided into four periods of analysis of eight years each. Eight years is equivalent to thirteen revolutions of *Venus*, or five synodic periods. For each period of analysis corrections to the mean longitude, eccentricity, and perihelion of both the Earth and *Venus* are obtained. A correction of about 3″ to Le Verrier's centennial motion of *Venus* is distinctly indicated, but a period of thirty-two years is naturally too short to obtain reliable corrections proportional to the time. I have therefore merely taken means for the four periods of analysis, and thus obtained corrections applicable to Le Verrier's elements for 1885·0.

The peculiarity of my analysis is that it contains no simultaneous equations, a feature which I believe will be appreciated by readers of Le Verrier, vol. vi. pp. 58–70, or Newcomb, *Astronomical Constants*, p. 92. This has been achieved by forcing symmetry into the distribution of observations by giving enhanced weight, whenever necessary, to isolated observations. The process no doubt slightly diminishes the weight of the results as compared with an ideal method, but, as I shall now show, it involves a considerable increase of accuracy as compared with the more laborious methods just referred to. In Newcomb's *Astronomical Constants*, p. 57, we read: "In dealing with so extensive a system of unknown quantities it is impracticable to investigate the dependence of each upon all the others." Now it is just this dependence, which Newcomb found it impracticable to investigate, which is brought out in the present paper. And the investigation is by no means unnecessary; for I shall show that the corrections to the eccentricity and perihelion of the Earth are determined by a vector quantity whose components are $y+z$, $y'+z'$. A similar statement holds for *Venus* with components $y+2z$, $y'+2z'$. It also appears that y, y' are quantities that can only be determined with 10 per cent. of the weight

of z, z'. Now when the dependence of each of the unknown quantities upon the others is not investigated, the errors of *Venus* are determined with no greater weight than y, y'. In the present analysis it is practicable to adopt the value of the vector $y+z$, $y'+z'$ directly from the observations of the Sun, and to pass to *Venus* by adding the well-determined vector z, z', thus obtaining $y+2z$, $y'+2z'$. Owing to the enormous number of observations available, there is no great difference between the numerical results of the two processes.

It is necessary to exhibit results by resolving into corrections to eccentricity and perihelion. This can be done, however, after the analysis; for the purposes of analysis it is preferable to resolve towards and at right angles to the direction of conjunction. During any period of analysis therefore, V, E will denote the mean longitudes of *Venus* and Earth measured from the common longitude of the third (or middle of five) inferior conjunction.

Period of Analysis.	Longitude of Third Inferior Conjunction.	Date.
1869–1876	222°·5	1873 May 4
1877–1884	220·2	1881 May 1
1885–1892	217·9	1889 April 29
1893–1900	215·6	1897 April 26

The variables $x\ x'$, $y\ y'$, $z\ z'$ are defined thus: the corrections to the longitudes of *Venus* and Earth are

$$\delta V = x+x'+2(y+2z)\sin V+2(y'+2z')\cos V$$
$$\delta E = \quad x'+2a(y+z)\sin E+2a(y'+z')\cos E$$

then, in order to conform to Kepler's laws, it is necessary to put for the variations of the radii vectores

$$\delta r' = -a(y+2z)\cos V+a(y'+2z')\sin V$$
$$\delta r = -a(y+z)\cos E+a(y'+z')\sin E$$

where a = ratio of mean distances = 0·7233.

It will be noticed that if y, y' alone are different from zero the displacements of the two planets are equal and parallel at all conjunctions; the geocentric displacement of *Venus* is therefore zero; y, y' therefore must be determined from observations remote from conjunction; the small weights with which they are determined may therefore be anticipated.

In forming the equations of condition the utmost simplicity is desirable. Complex equations are laborious, both to form and to solve. At the worst, the modifications introduced for the sake of simplicity will only cause the solution obtained to be an approximation to, instead of identical with, the solution of accurate equations. The approximate solution could be carried accurately into the tabular places, and fresh residuals formed,

Period I. 1869–1876. Unit $0^s.01$.
Longitude at middle inferior conjunction = $222°.5$.
Date 1873 May 4.

Mean Error of Right Ascension of Venus.

V–E.	o.	1.	2.	3.	4.	5.	Sum.
180°	...	+19	(+12)	+ 2	(+ 8)	(+ 6)	+ 47
195	...	+ 2	+ 9	+ 9	+ 7	− 3	+ 24
210	...	+13	(+11)	+ 6	+10	+ 3	+ 43
225	...	+29	+13	+14	+16	(+ 5)	+ 77
240	...	− 8	+ 9	+22	+16	(+ 6)	+ 45
255	...	+ 7	+21	(+15)	+ 7	+10	+ 60
270	...	+10	+ 2	+13	+12	+ 9	+ 46
285	...	+ 5	+19	+15	+16	+14	+ 69
300	...	+ 1	(+10)	+ 8	+ 5	+18	+ 42
315	...	− 4	+10	+ 6	+22	+23	+ 57
330	...	0	+10	+ 8	+11	+22	+ 51
345	...	− 7	+ 8	+ 8	+ 8	+15	+ 32
0	...	−13	−18	− 2	− 6	− 6	− 45
15	...	−10	− 9	− 4	(− 6)	+ 5	− 24
30	...	− 4	− 6	+ 5	− 8	− 4	− 17
45	...	− 2	− 1	− 3	+ 2	+10	+ 6
60	...	0	+ 2	+ 3	+ 4	+ 7	+ 16
75	...	+ 4	+ 3	+ 4	+ 4	+ 9	+ 24
90	...	+ 5	− 1	+10	+12	+ 3	+ 29
105	− 9	+12	− 5	+ 8	+ 1	...	+ 7
120	− 8	+ 7	+ 1	+ 3	+15	...	+ 18
135	+ 2	+ 7	+ 6	+15	+ 5	...	+ 35
150	(+ 1)	+14	+ 9	0	+ 8	...	+ 32
165	0	+14	+16	+10	+10	...	+ 50
					General Sum	... ...	+724

and a second approximation obtained with the simple equations of condition. The simplifications that I am about to introduce, however, do not appear to prevent the solutions from being accurate to within 5 or 10 per cent., and the imperfections of the observations themselves probably preclude a higher degree of accuracy. The errors of Le Verrier's tables are in fact so small that 5 per cent. of their amount is too small a quantity to determine. The simplifications are as follows :

In forming the variation of geocentric longitude in terms of the errors of heliocentric coordinates, the orbits are supposed circular and co-planar, the mean motions during one period of analysis are taken as accurately in the ratio 13 : 8, and an error

Period II. 1877–1884. Unit $0^{s}.01$.
Longitude at middle inferior conjunction = $220^{\circ}.2$.
Date 1881 May 1.

Mean Error of Right Ascension of Venus.

V−E.	0.	1.	2.	3.	4.	5.	Sum.
180°	...	(+ 2)	(0)	(+10)	(0)	(+ 6)	+ 18
195	...	+ 6	0	(+10)	− 1	(+ 6)	+ 21
210	...	+17	(+ 3)	+ 5	+10	+ 6	+ 41
225	...	+ 6	+ 7	+18	+22	+12	+ 65
240	...	+ 2	+ 8	+ 2	+20	(+ 6)	+ 38
255	...	+ 7	+16	+15	+ 8	+ 2	+ 48
270	...	+11	+14	+ 8	+ 4	+ 5	+ 42
285	...	+ 2	+ 6	+ 5	+ 5	+17	+ 35
300	...	+ 6	+ 4	+ 8	+12	+ 6	+ 36
315	...	− 1	+17	(+ 9)	− 1	+11	+ 35
330	...	+ 5	+10	+11	+ 5	+ 4	+ 35
345	...	− 8	+13	+ 3	−29	+13	− 8
0	...	− 2	+ 3	− 6	−55	+ 3	− 57
15	...	− 6	− 5	− 4	−34	+ 3	− 46
30	...	− 5	+ 1	+ 6	− 7	+ 6	+ 1
45	...	− 2	+ 2	+11	− 4	+ 7	+ 14
60	...	+ 8	+ 6	+13	+ 3	+ 5	+ 35
75	...	+ 7	+22	+ 7	+ 7	+15	+ 58
90	...	+ 1	+ 1	+ 9	+ 6	+ 9	+ 26
105	(+ 1)	+ 7	− 6	+ 6	+ 5	...	+ 13
120	0	+ 2	+12	+ 1	+ 8	...	+ 23
135	−18	+ 2	(+10)	0	+13	...	+ 7
150	− 2	+ 5	+ 8	− 6	+ 5	...	+ 10
165	(0)	0	+15	(0)	+ 7	...	+ 22
					General Sum	... ...	+512

of right ascension is treated as equivalent to an error of longitude.

We therefore put $V = 13\theta$ $E = 8\theta$ $V-E = \phi = 5\theta$.

For the geocentric distance Δ we have $\Delta^2 = 1 - 2a \cos\phi + a^2$.

For the variation of geocentric longitude

$$d\lambda = \frac{a}{\Delta^2}\left\{\left(-\frac{dr'}{a} + dr\right)\sin(V-E) + \left(a\,dV + \frac{dE}{a}\right) - (dV + dE)\cos(V-E)\right\}$$

whence substituting the values of dr' &c. in terms of x, x'...

Period III. 1885–1892. Unit $0^s{\cdot}01$.
Longitude at middle inferior conjunction = $217°{\cdot}9$.
Date 1899 April 29.

Mean Error of Right Ascension of Venus.

V–E.	0.	1.	2.	3.	4.	5.	Sum.
180°	...	+10	(+ 6)	(+10)	(+ 6)	(+ 7)	+ 39
195	...	+12	(+ 8)	+ 8	+ 6	+ 3	+ 37
210	...	+14	+10	−10	+ 5	+21	+ 40
225	...	+11	+14	+ 7	+12	+ 9	+ 53
240	...	+14	+12	+21	+11	+13	+ 71
255	...	+14	+17	+ 8	+ 5	+18	+ 62
270	...	+ 8	+12	+11	+ 5	+ 8	+ 44
285	...	+16	+ 9	+ 7	+ 4	+ 4	+ 40
300	...	+ 3	+11	+ 5	− 3	+10	+ 26
315	...	+ 4	+21	+ 2	+ 7	+ 8	+ 42
330	...	+ 4	+ 8	+15	− 4	+10	+ 33
345	...	0	− 9	+ 8	−23	+15	− 9
0	...	− 6	−30	−11	−55	+ 2	−100
15	...	− 7	−23	+ 1	−20	+15	− 34
30	...	− 1	−10	+10	− 6	+16	+ 9
45	...	− 1	0	+ 6	+ 2	+ 8	+ 15
60	...	+ 4	0	+ 9	+ 9	+20	+ 42
75	...	+ 8	+11	+14	− 9	(+15)	+ 39
90	...	+19	+ 9	+17	+ 4	+14	+ 63
105	(+ 6)	+14	+14	+ 8	+10	...	+ 52
120	(+ 3)	+ 4	+ 5	+12	+12	...	+ 36
135	+ 1	+ 8	−10	+11	+ 8	...	+ 18
150	− 3	+ 3	+14	+ 6	+10	...	+ 30
165	− 3	+ 5	+11	+ 8	(+ 8)	...	+ 29
				General Sum	...	...	+677

$d\lambda = ax + a'x + by + b'y' + cz + c'z' = \varepsilon =$ tab.–obs. error of R.A.

where $a = \frac{\alpha}{\Delta^2}\{\alpha - \cos 5\theta\}$

$a' = 1$

$b = \frac{\alpha}{\Delta^2} \sin 5\theta\{-\cos 13\theta + \alpha \cos 8\theta\}$

$b' = \frac{\alpha}{\Delta^2} \sin 5\theta\{\sin 13\theta - \alpha \sin 8\theta\}$

Period IV. 1893–1900. Unit $0^{s}\cdot01$.
Longitude at middle inferior conjunction $= 215^{\circ}\cdot6$.
Date 1897 April 26.

Mean Error of Right Ascension of Venus.

V–E.	0.	1.	2.	3.	4.	5.	Sum.
180°	...	(+13)	(+13)	(+15)	+18	(+15)	+ 74
195	...	+16	(+16)	+10	+ 9	+17	+ 68
210	...	+16	+19	+ 9	+ 7	+16	+ 67
225	...	+15	+12	+ 4	+16	+12	+ 59
240	...	+ 4	+14	+10	+25	+25	+ 78
255	...	+13	+15	+12	+ 7	+ 9	+ 56
270	...	+11	+17	+12	+ 2	+14	+ 56
285	...	+10	+13	+ 8	+ 6	+10	+ 47
300	...	+10	+11	(+ 7)	+10	+ 7	+ 45
315	...	+ 8	+12	+ 6	+11	+11	+ 48
330	...	− 2	+11	+ 8	−10	+12	+ 19
345	...	−12	+ 6	+ 6	−16	+16	0
0	...	−23	−14	0	35	+18	− 54
15	...	−14	0	+ 1	−20	+13	− 20
30	...	− 6	− 1	+16	− 4	+15	+ 20
45	...	− 6	+13	+19	+ 5	+14	+ 45
60	...	+ 7	+18	+21	+15	+17	+ 78
75	...	+20	+ 2	+19	+ 5	+20	+ 66
90	...	+21	+12	+12	+ 8	+11	+ 64
105	...	+20	+ 5	+13	+11	+ 7	+ 56
120	(+10)	+11	+ 6	+ 8	+ 9	...	+ 44
135	+ 5	+10	+12	+12	+17	...	+ 56
150	+ 2	+13	+10	+14	+11	...	+ 50
165	+11	+11	+15	−11	+13	...	+ 39
				General Sum	...	...	1061

$$c = \frac{a}{\Delta^2}[-\sin 18\theta + \tfrac{5}{2}a \sin 13\theta - \sin 8\theta - \tfrac{1}{2}a \sin 3\theta]$$

$$c' = \frac{a}{\Delta^2}[-\cos 18\theta + \tfrac{5}{2}a \cos 13\theta - \cos 8\theta - \tfrac{1}{2}a \cos 3\theta]$$

The mean values of aa', ab, ab' are all zero. Therefore, when symmetry has been imported into the observations by suitably manipulating the weights, the normal equations are

$$x = \Sigma a\epsilon \div \Sigma a^2 \text{ with a weight } \Sigma a^2 \text{ \&c.}$$

We now form tables from the preceding formulæ.

TABLE I.

Value of a. *Unit* = 0·01. *Mean Square* = 0·55.

V−E.	*a*.
180	+ 42
195	+ 42
210	+ 41
225	+ 41
240	+ 39
255	+ 37
270	+ 34
285	+ 29
300	+ 20
315	+ 2
330	− 38
345	−140
0	−261
15	−140
30	− 38
45	+ 2
60	+ 20
75	+ 29
90	+ 34
105	+ 37
120	+ 39
135	+ 41
150	+ 41
165	+ 42

The quantities in Tables I.–VI. are set down to two places of decimals, but they are subsequently used to one place only.

TABLE II.

$a' = 1{\cdot}0$. *Mean Square* = 1·0.

$$\frac{1-a^2}{\Delta^2} = 1 + 2\Sigma_1^\infty a^j \cos j\phi$$

$$\frac{(1-a^2)^3}{\Delta^4} = 1 + a^2 + 2\Sigma_1^\infty a^j \cos j\phi[(j+1)-(j-1)a^2]$$

whence the following mean values may be obtained of

$$\frac{1-a^2}{\Delta^2} = 1 \qquad \frac{(1-a^2)\cos 2\phi}{\Delta^2} = a^2$$

$$\frac{(1-a^2)\cos\phi}{\Delta^2} = a \qquad \frac{(1-a^2)\cos^2\phi}{\Delta^2} = \tfrac{1}{2}(1+a^2)$$

TABLE III.

Value of b. *Unit* 0·01. *Mean Square* = 0·13.

V−E.	1.	2.	3.	4.	0 or 5.
180	0	0	0	0	0
195	− 9	+ 4	+ 3	− 8	+ 10
210	− 9	− 4	+ 16	− 22	+ 19
225	+ 5	− 22	+ 31	− 29	+ 15
240	+ 28	− 41	+ 39	− 21	− 4
255	+ 49	− 48	+ 28	+ 2	− 32
270	+ 55	− 34	0	+ 34	− 55
285	+ 40	− 2	− 37	+ 62	− 63
300	+ 3	+ 39	− 65	+ 67	− 43
315	− 41	+ 68	− 69	+ 44	− 2
330	− 68	+ 65	− 37	− 5	+ 45
345	− 42	+ 15	+ 17	− 44	+ 53
0	0	0	0	0	0
15	− 53	+ 44	− 17	− 15	+ 42
30	− 45	+ 5	+ 37	− 65	+ 68
45	+ 2	− 44	+ 69	− 68	+ 41
60	+ 43	− 67	+ 65	− 39	− 3
75	+ 63	− 62	+ 37	+ 2	− 40
90	+ 55	− 34	0	+ 34	− 55
105	+ 32	− 2	− 28	+ 48	− 49
120	+ 4	+ 21	− 39	+ 41	− 28
135	− 15	+ 29	− 31	+ 22	− 5
150	− 19	+ 22	− 16	+ 4	+ 9
165	− 10	+ 8	− 3	− 4	+ 9

$$\frac{\sin^2\phi}{\Delta^2} = \tfrac{1}{2}$$

$$\frac{(1-a^2)^3}{\Delta^4} = 1 + a^2$$

$$\frac{(1-a^2)^3\cos\phi}{\Delta^4} = 2a$$

$$\frac{(1-a^2)^3\cos 2\phi}{\Delta^4} = a^2(3-a^2)$$

$$\frac{(1-a^2)^3\cos 3\phi}{\Delta^4} = a^3(4-2a^2)$$

$$\frac{(1-a^2)^3\cos^2\phi}{\Delta^4} = \tfrac{1}{2}(1+4a^2-a^4)$$

$$\frac{(1-a^2)\sin^2\phi}{\Delta^4} = \tfrac{1}{2}$$

$$\frac{(1-a^2)^3\cos 3\phi}{\Delta^4} = \frac{a}{2}(3+2a^2-a^4)$$

$$\frac{(1-a^2)\cos\phi\sin^2\phi}{\Delta^4} = \frac{a}{2}$$

Considerations of symmetry will show that the mean values of twelve out of the fifteen quantities aa' &c. are zero.

The above formulæ will readily show that aa', bc, $b'c'$ also take zero mean values.

TABLE IV.

Value of b'. Unit 0·01. *Mean Square* = 0·13.

V−E.	1.	2.	3.	4.	0 or 5.
180	0	0	0	0	0
195	+ 6	−10	+10	− 6	+ 1
210	+20	−21	+15	− 2	−11
225	+32	−23	+ 5	+14	−28
240	+32	− 9	−17	+36	−42
255	+14	+18	−43	+51	−40
270	−18	+47	−58	+47	−18
285	−52	+65	−54	+22	+18
300	−70	+58	−24	−19	+55
315	−59	+24	+21	−57	+72
330	−17	−26	+59	−70	+53
345	+32	−51	+50	−30	− 1
0	0	0	0	0	0
15	− 1	−30	+50	−51	+32
30	+53	−70	+59	−26	−17
45	+72	−57	+21	+24	−59
60	+55	−19	−24	+58	−70
75	+18	+22	−54	+65	−52
90	−18	+47	−58	+47	−18
105	−40	+51	−43	+18	+14
120	−42	+36	−17	− 9	+32
135	−28	+14	+ 5	−23	+32
150	−11	− 2	+15	−21	+20
165	+ 1	− 6	+10	−10	+ 6

We also obtain

mean value of $a^2 = \frac{1}{2}\,\frac{a^2}{1-a^2}$

,, ,, $a'^2 = 1$

,, ,, b^2 or $b'^2 = \frac{1}{4}a^2$

,, ,, c^2 or $c'^2 = \frac{a^2(1+\frac{1}{4}a^2)}{1-a^2}$

The observations are divided into groups of 24·33006 days each ; the centre of each group corresponds to an exact multiple of 15° in the mean heliocentric elongation. There are 120 groups in each period of analysis. The mean error of right ascension is given in the following tables. Whenever an error is enclosed in brackets, it implies there were no observations in the corresponding group, but an error has been interpolated from

TABLE V.

Value of c. *Unit* 0·01. *Mean Square* = 1·24.

V−E.	1.	2.	3.	4.	0 or 5.
180	0	+ 49	− 80	+ 80	− 49
195	− 57	+ 83	− 78	+ 42	+ 9
210	− 87	+ 77	− 37	− 17	+ 65
225	− 77	+ 32	+ 24	− 72	+ 92
240	− 30	− 30	+ 78	− 97	+ 78
255	+ 33	− 82	+101	− 81	+ 30
270	+ 83	−103	+ 83	− 32	− 32
285	+101	− 86	+ 37	+ 26	− 79
300	+ 86	− 46	− 11	+ 64	− 92
315	+ 60	− 21	− 27	+ 64	− 77
330	+ 86	− 59	+ 10	+ 43	− 80
345	+271	−216	+ 78	+ 90	−224
0	+497	−308	0	+308	−497
15	+224	− 90	− 78	+216	−271
30	+ 80	− 43	− 10	+ 59	− 86
45	+ 77	− 64	+ 27	+ 21	− 60
60	+ 92	− 64	+ 11	+ 46	− 86
75	+ 79	− 26	− 37	+ 86	−101
90	+ 32	+ 32	− 83	+103	− 83
105	− 30	+ 81	−101	+ 82	− 33
120	− 78	+ 97	− 78	+ 30	+ 30
135	− 92	+ 72	− 24	− 32	+ 77
150	− 65	+ 17	+ 37	− 77	+ 87
165	− 9	− 42	+ 78	− 83	+ 57

neighbouring observations. It will be seen that bracketed errors are most frequent at superior conjunction, but seldom occur elsewhere.

The analysis of the errors consists simply in multiplying each entry by the corresponding entry of Tables I.–VI. and dividing by 120 times the mean square of the factor.

For example, Period I. 1869–1876, to find x.

The formula is $x = \Sigma a\varepsilon \div \Sigma a^2$, where ε denotes an entry in the table of Period I., and the values of a are given in Table I. As the value of a is a function of V−E only, we naturally proceed to add horizontally (the sums are exhibited in the extreme right-hand column).

Similarly $x' = \Sigma a'\varepsilon \div \Sigma a'^2 = \dfrac{\Sigma\varepsilon}{120}$, since $a' = 1$. The value of $\Sigma\varepsilon$ is exhibited in the right-hand bottom corner.

TABLE VI.

Value of c'. *Unit* 0·01. *Mean Square* = 1·24.

V−E.	1.	2.	3.	4.	0 or 5.
180	− 84	+ 68	− 26	− 26	+ 68
195	− 63	+ 18	+ 34	− 74	+ 85
210	− 10	− 43	+ 80	− 86	+ 59
225	+ 51	− 86	+ 89	− 57	+ 4
240	+ 92	− 92	+ 57	0	− 57
255	+ 95	− 58	− 2	+ 61	− 96
270	+ 61	0	− 61	+ 98	− 98
285	+ 6	+ 55	− 94	+ 98	− 64
300	− 39	+ 82	− 93	+ 69	− 19
315	− 48	+ 74	− 72	+ 43	+ 3
330	− 18	+ 65	− 88	+ 77	− 37
345	− 6	+166	−261	+256	−153
0	−161	+423	−523	+423	−161
15	−153	+256	−261	+166	− 6
30	− 37	+ 77	− 88	+ 65	− 18
45	+ 3	+ 43	− 72	+ 74	− 48
60	− 19	+ 69	− 93	+ 82	− 39
75	− 64	+ 98	− 94	+ 55	+ 6
90	− 98	+ 98	− 61	0	+ 61
105	− 96	+ 61	− 2	− 58	+ 95
120	− 57	0	+ 57	− 92	+ 92
135	+ 4	− 57	+ 89	− 86	+ 51
150	+ 59	− 86	+ 80	− 43	− 10
165	+ 85	− 74	+ 34	+ 18	− 63

The details of the calculation of y, y', z, z' are not exhibited: they consist merely in 120 multiplications. The factors used are given in Tables III., IV., V., and VI., but one less significant figure is employed.

Multiplying by 15, to transform to seconds of arc, we thus get the following table of results:

Period.	Long. of Conjunction.	Date.	Wt. 55. x	Wt. 100. x'	Wt. 13. y	Wt. 13. y'	Wt. 124. z	Wt. 124. z'
I.	222·5	1873 May 4	+0″·75	+0″·90	+0″·44	0″·00	−0″·23	−0″·15
II.	220·2	1881 1	+0·90	+0·64	+0·55	+0·18	−0·57	−0·41
III.	217·9	1889 Apr. 29	+1·27	+0·84	+0·54	+0·15	−0·33	−0·61
IV.	215·6	1897 26	+1·15	+1·32	+0·84	−0·22	−0·60	−0·42
Means	219 0		+1·02	+0·92	+0·59	+0·03	−0·43	−0·40

This completes the analysis: it is impossible to get reliable terms proportional to the time from thirty-two years' observations, but the run in the values of $x+x'$, the correction to the mean longitude of *Venus*, approximately confirms Newcomb's diminution by 3″ of Le Verrier's centennial motion.

The mean results apply to the date 1885·0.

From the Sun observations I obtained as a correction to the Sun's longitude

$$\text{(tab.—obs.)} \quad +0''\cdot78-0''\cdot47\sin(E-115^\circ)$$
$$\text{or} \quad +0\cdot78-0\cdot46\cos(E-219)+0''\cdot11\sin(E-219^\circ)$$

Dividing the last two coefficients by $2a = 1\cdot447$, we have from the Sun observations and in the notation of this paper

$$x' = +0''\cdot78 \quad y+z = +0''\cdot08 \quad y'+z' = -0''\cdot31 \qquad \ldots \text{ (i.)}$$

These values are entitled to far greater weight than the corrections to the solar elements obtained from *Venus*

$$x' = +0''\cdot92 \quad y+z = +0''\cdot16 \quad y'+z' = -0''\cdot37 \qquad \ldots \text{ (ii.)}$$

It will be seen, however, that the accordance is most satisfactory.

In passing to the corrections for *Venus* I use values (i.) and reject values (ii.) for x', $y+z$, $y'+z'$. This makes it possible to get corrections for *Venus* independently of the values of y, y'.

We thus have for *Venus*

$$x+x' = +1''\cdot80 \quad y+2z = -0''\cdot35 \quad y'+2z' = -0''\cdot71$$

or Le Verrier's longitude of *Venus* requires corrections in 1885·0,

$$-1''\cdot80+2\times0''\cdot35\sin(V-219^\circ)+2\times0''\cdot71\cos(V-219^\circ)$$
$$=-1\cdot80+2\times0\cdot72\sin(V-130)-2\times0\cdot34\cos(V-130),$$

on resolving in the direction of the perihelion of *Venus*.

Hence corrections required by Le Verrier's elements in 1885·0 are

Element.	Correction required by Le Verrier in 1885·0.
Mean longitude	$-1''\cdot80$
Eccentricity	$+0\cdot72$
Coefficient of principal elliptic term in longitude	$+1\cdot44$
Mean anomaly	$-0\cdot34\div e = -50''$
Perihelion	$+48''$

It is interesting to compare these corrections with the differences between Le Verrier's tables and Newcomb's.

Counting T in centuries from 1900·0 we have the following differences, Newcomb–Le Verrier :

Formula.

Element.	For Difference Newcomb–Le Verrier.	Value in 1885·0.
Mean longitude ...	$-1''\cdot74-3''\cdot29T-0''\cdot0141T^2$	$-1''\cdot25$
Perihelion	$+83\cdot9+128\cdot66T+2\cdot415T^2$	$+64\cdot6$
Principal elliptic coefficient in longitude	$+1''\cdot79+2''\cdot518T-0''\cdot015T^2$	$+1''\cdot41$

There is thus most satisfactory agreement between the eccentricity found in this paper and that used by Newcomb. The mean longitude and perihelion cannot be compared unless allowance is made

(i.) For any changes of long-period terms between Le Verrier's tables and Newcomb's.

(ii.) For the difference o″·8 between Newcomb's equinox and that used in the Greenwich clock-star list up to and including 1902.

It will be seen that (ii.) approximately explains the difference between Newcomb's correction to the mean longitude and that obtained in this paper.

It will be seen that the analyses of this paper are unaffected by the following alterations which involve odd functions of the elongation only in the geocentric places :

(i.) A change in Le Verrier's mass of the Earth + Moon which will introduce corrections varying as sin (V − E), sin 2(V − E), &c. with the tabular geocentric places.

(ii.) A change in the tabular semidiameter and Stone's empirical alteration thereof, which, as it takes opposite signs before and after conjunction, can presumably be expanded in a form

$$\pm\mu+a_1\sin(V-E)+a_2\sin 2(V-E)+\ \ldots$$

I have not yet examined how far it is possible to disentangle errors of defective illumination from errors due to Le Verrier's mass of the Earth + Moon, and so obtain a determination of this mass ; but it is fairly obvious that no such determination can compete with the value resulting from pendulum experiments.

It is sufficient for the purposes of this paper to point out that the mean value of the product of either of the above two corrections by any of the six quantities a, a', b, b', c, c' is by symmetry zero.

Comparison of Carrington's Circumpolar Catalogue with the Greenwich-Groombridge System. By W. G. Thackeray.

(*Communicated by the Astronomer Royal.*)

The forthcoming Greenwich Catalogue for 1900 will consist of two parts : Part I., Fundamental and Zodiacal Stars ; Part II., Astrographic Reference Stars. This latter part consists of some 10,000 circumpolar stars within 26° of the pole. As the Carrington Catalogue affords the only data from which the proper motion of the majority of the stars within 9° of the pole can be determined, it is necessary to investigate its systematic errors, and for this purpose a comparison has been made with those stars which are common to it and the Groombridge-Greenwich system. There are ninety-four stars available, which have been grouped in degrees according to the Carrington Catalogue place.

The Carrington Catalogue contains 3735 stars, extending from 0° to 9° polar distance. The positions are based on instrumental errors derived from twenty special polars the places of which were finally determined by reference to double transits of *Polaris* and *Ursæ Minoris* (Introduction, pp. 13, 22–35). Four or more of these stars—two above pole and two below pole whenever possible—were taken for the determination of the meridian error and of the polar point (p. 15). The circle, which was made to turn on its collar and which was divided by Troughton & Simms's well known engine, was not provided with any means for the determination of errors of division. As the circle was not moved by Carrington during the period covered by his catalogue observations the division errors are the same throughout and necessarily remain uncorrected (p. 16). There were four fixed microscopes with which to read the circle, a vertical and horizontal pair, but after the first year only the horizontal pair were used (p. 7). The catalogue stars were observed and arranged in three sub-zones.

The first	sub-zone	from	45′	to	4°	of N.P.D.
„ second	„	„	4°	„	7°	„
„ third	„	„	7	„	9	„

The places of the stars in the first sub-zone depend on practically an equal number of observations above and below pole. The stars in the second sub-zone for 4° to 6° are mostly observed for the first three hours either one above or two below, or two above and one below. For 7°, with few exceptions, and for the third sub-zone, stars are observed entirely above pole.

The results of the comparison taken as corrections to Carrington's Catalogue place given in the following tables are :—

	0°.	1°.	2°.	3°.	4°.	5°.	6°.	7°.	8°.	9°.
	s	s	s	s	s	s	s	s	s	s
R.A.	+0·7	+0·4	−0·3	−0·1	+0·2	−0·4	−0·3	−0·2	−0·4	0·0
	"	"	"	"	"	"	"	"	"	"
N.P.D.	−0·3	+0·3	+0·5	+0·6	+0·9	+1·0	+1·1	+1·2	+1·4	+0·1
No. of stars	1	7	4	11	10	15	11	11	17	7

It has been decided to apply a systematic correction of $-0^s\cdot3$ to the right ascension of stars of 5° to 9° N.P.D. The corrections to the north polar distances which are regular and systematic seem to point to the influence of uncorrected division errors, and this is confirmed from further comparing the observations above with those below pole, which can be easily done from the ledgers in the sub-zones where the observations below the pole are marked with an asterisk (*) and the difference from the adopted catalogue place given for each observation.

The following is the excess of N.P.D. above pole :—

0°.	1°.	2°.	3°.	4°.	5°.	6°.	7°.
"	"	"	"	"	"	"	"
−0·1	−0·3	−0·4	−1·0	−1·0	−1·3	−1·4	−1·6
12	122	178	302	104	95	121	6

The stars at 4° are made up of thirty-nine stars in the first sub-zone and sixty-five in the second sub-zone.

The following are the systematic corrections to be adopted for the north polar distances.

For the second sub-zone in which the observations below pole are not symmetrical and are most numerous at 0^h–3^h, in order to obviate systematic differences it will be first necessary to bring into line the catalogue place of all stars which have S.P. observations by applying a correction to all S.P. observations as follows :—

N.P.D.	Correction.
°	"
4	−1·0
5	−1·3
6, 7°	−1·4

Then, as the first sub-zone contains an equal number of observations above and below pole, and in the third sub-zone all the observations are above pole, the following are adopted as the corrections from the catalogue comparison :—

N.P.D.	Correction.	N.P.D.	Correction.
° °	"	°	"
0, 1	+0·3	6	+1·1
2	+0·5	7	+1·2
3	+0·6	8	+1·2
4	+0·9	9	+1·2
5	+1·0		

Comparison between Carrington and Groombridge-Greenwich in R.A. for each 1° of N.P.D.

0°.		1°.		2°.		3°.		4°.		5°.		6°.		7°.		8°.		9°.	
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s
0·7*		0·2*		0·3			1·4		0·9		0·5		0·2		0·0		0·2		0·1
	0·3*		0·6*			0·6		0·2			0·7		0·9	0·3		0·3			0·5
	0·1*		1·0	0·1			0·3*			0·5		0·1		0·1			0·2		0·5
	0·2*		0·0*	0·3*			0·6*				0·4		1·1		0·1	0·1		0·2	
	0·7		...			0·5*		0·0			0·8		1·2		1·4		0·2		0·2
	0·6*		...		0·4			0·8			0·6	0·1			1·0		0·6	0·5	
	1·5		...			0·1		0·5			0·4	0·4			0·2		0·3	0·4	
	...		...		0·1*				0·6		0·3		1·1		0·0		0·3		...
	...		...		0·7			0·4			0·4	0·1			0·5		0·6		...
	...		...			0·1*		0·2			1·0		0·0	0 3			0·5		...
	...		...		0·1*			...			0·8	0·5			0·1		0·8		...
	...		...		...			...			0·6		...		...		0·9		...
	...		...		...			...			0·1		...		...		1·2		...
	...		...		...			...		0·4			...		...		0·6		...
	...		...		...			...		0·0			...		...		0·0		...
	...		...		...			...		...			...		...		0·2		...
	...		...		...			...		...			...		...	0·2			...
Means		+0·4		−0·3		−0·1		+0·2		−0·4		−0·3		−0·2		−0·4		0·0	

Comparison between Carrington and Groombridge-Greenwich in N.P.D. for each 1° of N.P.D.

0°.		1°.		2°.		3°.		4°.		5°.		6°.		7°.		8°.		9°.	
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
	0·3*	1·2*		0·7			0·1	0·9		1·2		0·5			0·3	1·7		1·4	
		0·1*		0·4*			0·4	0·8		1·8		1·3		1·7		0·8		0·4	
		0·2*		0·7		1·1		0·8*		0·5		0·4		1·7		1·6		0·5	
		0·9*		0·3*		0·7*		0·7*		1·8		1·5		2.3		1·6		1·2	
			0·3	...		1·1*		0·3		0·9		1·1		1·0		0·4		0·9	
		0·3*		...		1·4		1·2		0 6		1·4		1·0		1·1		0·9	
		...		...		1·2		1·9		1·0		1·3		1·6		1·5		1·4	
		...		...		0·2*		1·1		1·1		1·0			0·6	0·7		...	
		...		...		0·2		0·4		0·6		1·3		2·8		1·2		...	
		...		...		0·5*		1·2		1·3		1·2		1·2		2·6		...	
		...		...		0·4*		...		0·9		0·8		0·5		1·5		...	
		...		...		...		...		0·9		...		...		2·0		...	
		...		. .		...		...		1·0		...		...		1·5		...	
		...		...		...		...		1·5		...		...		1·7		...	
		...		...		...		...		0·3		...		...		2·0		...	
		...		...		...		...		...		...		...		1·4		...	
		...		...		...		...		...		...		...		0·4		...	
Means		+0·3		+0·5		+0·6		+0·9		+1·0		+1·1		+1·2		+1·4		+1·0	

Reference Numbers from Carrington's and Groombridge's Catalogues of the Comparison Stars for each 1° of N.P.D.

0°.		1°.		2°.		3°.		4°.		5°.		6°.		7°.		8°.		9°.	
Carr.	Groomb.	Carr.	Groomb.	Carr.	Groomb.	Carr.	Groomb.	Carr.	Groomb.	Carr.	Groomb.	Carr.	Groomb.	Carr.	Groomb.	Carr.	Groomb.	Carr.	Groomb.
1035*	1119	117*	144	140	175	145	195	52	67	451	595	1538	1633	593	774	76	100	346	506
		181*	235	956*	1141	236	339	131	177	1127	1359	2080	2063	1165	1391	1021	1255	429	580
		1834*	1884	1830	1871	491	642	575*	750	1516	1620	2105	2079	1222	1431	1045	1278	675	856
		1972*	2006	2316*	2283	842*	1004	770*	944	1746	1818	2222	2170	1807	1858	1301	1480	2348	2275
		2048	2065	...	...	1793*	1850	782	956	1790	1848	2261	2196	2360	2286	1381	1537	2350	2276
		3058*	3308	...	...	2122	2099	1205	1418	1818	1860	2404	2315	2550	2422	1558	1643	3140	3268
		3138	3402	...	...	2258	2210	2001	2007	1836	1879	2648	2476	3323	3597	1580	1662	3149	3276
						2755*	2628	3441	3820	1843	1889	2729	2548	3387	3707	1698	1778		
						2771	2667	3444	3824	1847	1892	2820	2708	3389	3709	1707	1782		
						3273*	3548	3693	4193	1882	1923	2822	2712	3502	3928	1774	1842		
						3621*	4101	...	...	1913	1937	3525	3970	3681	4174	1784	1843		
										1914	1940	...	...	...	...	1868	1909		
										2271	2213	...	...	...	...	1889	1927		
										3091	3212	...	...	...	...	1968	1977		
										3123	3260	...	...	...	...	2088	2071		
																3133	3261		
																3189	3370		

An asterisk (*) denotes that the star was one of Carrington's special polars.

The Total Solar Eclipse of 1908 *January* 3.

By A. M. W. Downing, D.Sc., F.R.S.

There are two Pacific islands favourably placed for the observation of this eclipse, viz. Hull Island, one of the Phœnix group, and Flint Island, one of the Line group. The astronomical details of the eclipse for these two islands are given below, the calculations having been made from the data of the *Nautical Almanac*. As the errors of the Moon's tabular places now amount to sensible, and apparently increasing, quantities, observers are warned that the calculated times of the various phases of the eclipse may differ considerably from the observed times, the calculated times being (with the present values of the errors) too late. In order to obviate, as much as possible, inconvenience arising from this cause, I have added in each case the intervals in time from the instant when the cusps subtend an angle of certain specified value at the Sun's centre to the commencement of totality.

The plans of the islands here reproduced are taken from the official charts published by the Hydrographic Office. The geographical details are abridged from the *Admiralty Sailing Directions*, *Pacific Islands*, vols. ii. and iii., 3rd ed., 1900.

The offices of Levers' Pacific Plantations Company, mentioned below, are situated at Port Sunlight, Cheshire. Intending observers of the eclipse should communicate with the Company at this address.

My thanks are due to Capt. A. M. Field, R.N., F.R.S., for his kindness in referring me to the available sources of information regarding these islands.

HULL ISLAND. Long. 172° 13′ W. Lat. 4° 30′ S.

Mean Solar Time.

	Greenwich.	Local.	Angle from North Point.	Vertex.
	d h m s	d h m s	°	°
First Contact. ...	Jan. 3 7 18 33	Jan. 2 19 49 41	281	5
Total Eclipse ...	3 8 30 36	2 21 1 44	88	162
	3 8 33 27	2 21 4 35	294	8
Last Contact ...	3 9 58 12	2 22 29 20	100	155

Duration of totality, 2m. 51s.

Sun's altitude at totality, 43°.

Angle of Cusps.	Time before Commencement of Totality.
°	s
90	34
60	13
45	7
30	3
15	¾

FLINT ISLAND. Long. 151° 48′ W. Lat. 11° 26′ S.

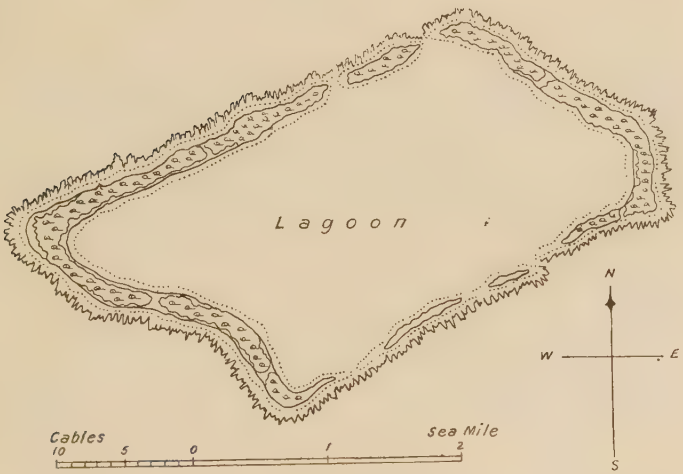
Mean Solar Time.

	Greenwich.				Local.				Angle from North Point.	Vertex.
	d	h	m	s	d	h	m	s	°	°
First Contact. ...	Jan. 3	7	52	51	Jan. 2	21	45	39	278	355
Total Eclipse ...	3	9	22	44	2	23	15	32	107	153
	3	9	26	44	2	23	19	32	262	308
Last Contact. ...	3	11	2	59	3	0	55	47	90	42

Duration of totality, 4m. 0s.

Sun's altitude at totality, 74°.

Angle of Cusps.	Time before Commencement of Totality.
°	s
90	47
60	18
45	10
30	4
15	1



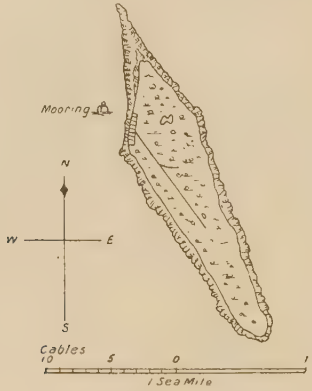
Hull Island.

HULL ISLAND (British) was discovered and so named by Wilkes in 1840. It has a lagoon and a little fresh water, and cocoanut trees on it, about 50 feet high. A coral reef fringes the island, and landing is difficult except by entering the lagoon by means of the boat passages on the north-west side. There is no anchorage.

In 1899 there were no inhabitants on the island.

It is now leased to Levers' Pacific Plantations Company.

The winds are almost constantly from the eastward, but squalls, accompanied by light rain, occur in the neighbourhood all the year round. The wind is variable from January to May, during which period bad weather is most common.



Flint Island.

Flint Island (British), discovered in 1801, is 13 feet high, covered with brushwood and trees, and is visible from the mast-head from a distance of 16 miles. It is about $2\frac{1}{2}$ miles long N.N.W. and S.S.E., half a mile wide, and is fringed by a steep coral reef which dries at low water, and extends seaward generally about half a cable, but off the northern end of the island it extends seaward $4\frac{1}{2}$ cables, and off the southern end E.S.E. $2\frac{1}{2}$ cables. In the interior are two small lagoons of brackish water.

In 1880 October the island was uninhabited, and the buoys formerly in use were gone.

It is now leased to Levers' Pacific Plantations Company.

There is little or no rise and fall of tide at Flint Island.

The landing is very bad even for surf-boats, but it is said to export nearly 200 tons of copra annually.

Preliminary Account of Flash Spectra taken 1905 *August* 30.
By S. A. Mitchell, Ph.D.

Upon the invitation of Admiral Chester, Superintendent of the U.S. Naval Observatory, the writer became a member of the United States expedition to observe the total eclipse of the Sun. The party crossed the Atlantic in the three men-of-war, U.S.S. "Minneapolis," U.S.S. "Cæsar," and U.S.S. "Dixie."

Observations were made at three stations, one in Africa, near the central line at Guelma, the other two in Spain, of which one was an edge station at Puerto Coeli, and the second near the centre of the Moon's shadow path at Daroca. All three stations were fortunate in having perfect weather conditions.

At Daroca were located Professor Eichelberger and Mr. Yowell, of the Naval Observatory; Professor Bigelow, of the U.S. Weather Bureau; Mr. Hoxton, of the Johns Hopkins University; and the writer. The programme of work included :—

A. Meteorological Observations, Professor Bigelow.

B. Photographs of Corona with 40-ft. horizontal camera, Mr. Hoxton.

C. Photographs of Corona with
(*a*) 104 in.
(*b*) 36 in. (5 in. aperture) } Mr. Yowell.

D. Spectroscopic work, Dr. Mitchell.

Five spectrographs were employed, three being grating instruments, the other two alike having a dispersion of one weak prism.

The results of the photographs are as follows :—

1. *Parabolic Grating.*—This spectrograph was used without a slit, and consisted of a Rowland grating—ruled, however, on a parabolic surface instead of on the usual concave—and a photographic plate. It was placed horizontally, and a light was fed to it by a transit of *Venus* cœlostat, which, however, did not work with the highest degree of precision. The grating was four inches in diameter, with 14,438 lines to an inch and a focal length of five feet. The exposures were made by Ensign A. G. Howe, U.S.N., of the "Minneapolis." The spectrum was exceedingly bright in the first order on one side, and definition was splendid. The flash extends from D_3 at the end of the long wave-lengths to λ 3300 in the ultra-violet.

The dispersion is practically the same as the three prism spectrographs of the Lick and Yerkes Observatories, so that the distance from D_3 to H is almost exactly seven inches, the total length of the photographed spectrum being 9·5 inches. The lines of the flash have not been counted, but they are probably more than 1500 in number. As the definition is excellent throughout and the spectrum is "normal" it is probable that wave-lengths will be determined with a high degree of accuracy.

The spectrum taken near mid-totality shows some interesting coronal rings. The ring in the green due to "coronium" is very plain, and besides there are two rings in the extreme ultra violet which are just as prominent on the photograph as the green ring. As the photographic action of the plate used is about as intense in the green as in the ultra-violet (λ 3400), where the sensitiveness falls off very quickly, it would seem that the corona is very rich in ultra-violet rays. The following lines are seen in the corona at wave-lengths which are approximately λλ 3381, 3388,

3455, 3643, 3984, 4228, 4565, 4618, and the "coronium" line at λ 5303.

2. *Flat grating*, with 15,000 lines per inch and a ruled surface of $3\frac{1}{2} \times 6$ inches. This was used with a Clark 5-inch visual lens of 6 feet focus, and was employed without a slit. The instrument placed horizontally in connection with a Gaertner cœlostat was used on the eclipse day by Dr. Mitchell, assisted by two sailors of the U.S.S. "Minneapolis." On the photograph the distance from D_3 to H is eight inches. In the flash spectrum lines can be seen beyond the D lines toward the red almost to C. This is the end of the spectrum most desired with this instrument, and the focus is excellent from F to the extreme of the red. The green "coronium" ring also appears on the plates taken near mid-totality.

3. *Short focus concave grating* with a ruled surface of 4×6 inches, with 3610 lines per inch, and a radius of curvature of 64 inches (focus 32 inches). On account of its small dispersion, the spectra were excessively bright, and it was intended to record the changes in the bright lines just before and just after totality by photographing the second and third orders of spectra on the same plate, and making a number of exposures in quick succession at second and third contacts, the times of exposures being recorded on the chronograph. This instrument was in charge of Ensign F. McCommon, U.S.N. However, as totality began about ten seconds before calculated time, the exposures at second contact which were to run on both sides of the "flash" were all inside totality. In addition, the chronograph pen failed to write, and, as no one was watching it, this was unnoticed till totality was over.

4. *Two prismatic spectrographs* of the same dispersion were used in an attempt to find the intensity of the corona in different parts of the spectrum. This was to be done by photographing on the same plate, above and below the corona, comparison spectra of the Sun taken under ordinary conditions and with different exposures. Photometric measures could then have been made.

The Smithsonian Institution bolometric determinations give the energy of the Sun in different parts of the spectrum. If these eclipse photographs had been successful the energy of the corona at different regions would have become known. The instruments were used with slits, and each consisted of one prism with the necessary collimator and objective. No photometric measures were possible, for the reason that the coronal spectra were very faint, caused no doubt by the plate being slightly fogged from the diffused light in taking the comparison spectra.

Columbia University, New York City:
1906 *February* 13.

Note on Mr. Reynolds's Dark-slides. By A. E. Conrady.

At the suggestion of Professor Turner I have collected all available information concerning the above in the hope that it may prove useful in future.

It appears that these slides were made from a log of Spanish mahogany originally cut into boards fifteen years ago, the wood having been kept by the makers of these slides for the last nine years, and used in the making of a large number of slides, none of which have ever shown any sign of fogging plates excepting those taken to Egypt by Mr. Reynolds.

When the slides were returned to the makers, plates were inserted, and the slides kept in a dark-room for about a week, when on development the plates were found decidedly fogged, excepting where a strip of tinfoil had been interposed between the wood and the film. It is thus evident that the slides were then capable of fogging plates even at a comparatively low temperature, and the experiment further showed that, whatever the emanation or radiation might be, it was incapable of acting through tinfoil. Unfortunately it was found quite impossible permanently to fasten tinfoil inside the shutters, but it was found that a heavy coating of the usual optical black, that is to say, essentially a mixture of shellac and lamp-black, also stopped the action entirely, and this was the method by which the slides were put right.

It must still remain an open question what caused these particular slides to acquire the undesirable peculiarity; possibly exposure to the extreme heat and dryness of Central Egypt may have had something to do with it, as it is certainly a fact that out of the large number of dark-slides made from this particular log Mr. Reynolds's slides were the only ones to show this property.

It would indeed seem advisable, when slides are to be used in the taking of photographs which cannot be replaced afterwards, severely to test them before use under as nearly as possible the conditions which they will have to face in the actual work.

I may add that these particular slides were not newly made for the eclipse; on the contrary they were manufactured two and a half years before, and had been in stock ever since.

Note on certain Anomalies observed in Radial Velocity Curves. By Alex. W. Roberts, D.Sc.

Those whose researches lead them in one direction of an inquiry regarding the causes which underlie short-period variation must be impressed, as well as oppressed, with the great area of uncertainty which surrounds the whole subject.

At first sight we seem to know practically nothing of the immediate circumstances which produce variation of the definite type to which such stars as *η Aquilæ* and *δ Cephei* belong.

And yet the uncertainty is neither complete nor final. We are convinced, for example, that revolution and variation, or it may be rotation and variation, are connected together in some intimate relation, and that a solution of the problem of short-period variation will be obtained when we are able to declare what the nature and extent of this relation is.

Any investigation, therefore, that purposes dealing with the measures of radial velocity obtained by spectroscopic observation must have a direct bearing on the wider problem of stellar variation. By considering the orbital movement of any binary system that also exhibits light pulsations, we are approaching this problem from a less difficult and probably a more hopeful direction.

It is as a contribution to variable star astronomy, rather than to spectroscopic research, that I offer this note on certain anomalies observed to exist in the velocity curves of variable stars of the short-period type.

If two stars whose dimensions are relatively very small compared with the size of the orbit they move in are bright enough to be observed spectroscopically, then their radial velocity curve will be of a very simple type.

The curve will reveal the orbital movement of the stars and nothing more. There will be no anomalies to consider.

If, however, the size of the stars is distinctly comparable with the size of their orbit—that is, if the components are near enough to mutually distort each other—then the radial velocity curve is no longer that due to the revolution of two practically distant globular particles round each other, but is instead the velocity curve of two contiguous ellipsoidal masses of matter—a somewhat different condition of things. In this latter case the observed radial velocity curve will be a sinuous line on either side of a mean curve computed on the assumption that the component stars are two luminous points.

Another grave disturbing element is introduced when the contiguous stars move in an eccentric orbit. It is evident that with eccentric movement there will be a corresponding rhythmic flux of figure. As the stars approach each other they will become more elongated; as they recede they will naturally tend to a more spherical form. This alternate contraction and expansion of the major axis of figure and the synchronous expansion and contraction of the minor axis must produce a secondary variation in the mean radial velocity curve, a variation depending largely upon the contiguity of the component stars, and the eccentricity of the orbit they are moving in. This secondary variation will also be considerably modified by the inclination of the orbit, the position of the line of apsides, and the density of the circling stars.

As our knowledge regarding the laws which govern the actual alteration in figure of gaseous masses is so uncertain, and the relation in such systems of pressure to form so complicated, we may leave this physical cause of secondary radial velocity undealt with until we have considered those more geometrical conditions of position and figure which, theoretically at least, produce a departure from the mean velocity curve.

With one of these conditions, viz. the ellipsoidal form of the component stars, we have a more sure groundwork of fact to build upon, and I purpose considering in a very general manner

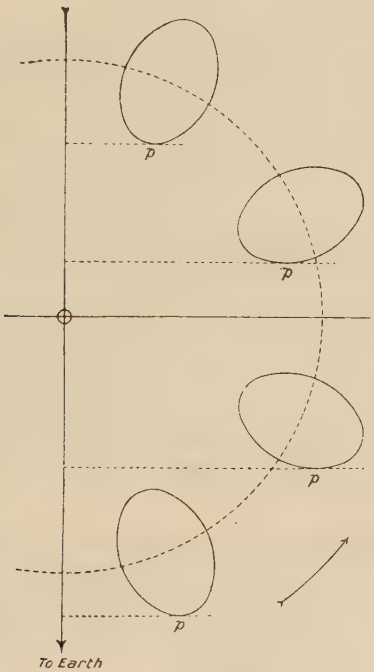


Fig. 1.

the character of the secondary sinuous curve produced by the revolution of such a body round the centre of gravity of its system.

It is evident that if the figure of a close binary star departs considerably from the spherical form, then the actual observed velocity curve will vary in a certain definite manner from the mean velocity curve. The reason for this is that the velocity recorded by the spectroscope is not that of a particle at the centre of the revolving star, but of an envelope of particles forming the outer shell of the prolate mass. If this shell be

spherical there will practically be no aberration in velocity ; but if it be ellipsoidal, or apioidal, then there will be a departure from the mean velocity curve the extent of which will be measured by the prolateness of the star.

To give a less abstract character to this note let us consider fig. 1 to represent a bright star revolving round a centre, O, in the direction indicated by the arrow. Let there be three planes of reference at right angles to one another : the tangent plane perpendicular to the line of sight, the vertical plane in which the line of sight lies, the horizontal plane containing the line of nodes. The intersection of the second and third planes is the line of sight, and all pass through the point O.

Now if fig. 1 represents four positions of the bright component, then the radial velocity of a particle at the point p, where the tangent plane or a parallel plane touches the circumference of the star, will more correctly define the true velocity curve of the system (considering its figure to be constant) than any other point within or on the outer surface of the star.

It will be evident that the velocity of the centre of the star will not come within the area of actual observation ; it has only a theoretical value and importance as furnishing the mean velocity curve on either side of which the apparent or true velocity curve varies. Since we do not see the centre we cannot regard the observed velocity curve as representing the velocity of this point. It represents the velocity of some other point or assemblage of points, and in my judgment it represents the velocity of p, the point where the tangent plane or a parallel plane touches the star.

Accepting this conclusion as a working hypothesis we may readily compute the amount of radial variation due simply to the prolateness of the component stars.

Let

θ = angular distance of bright star from line of sight at time T_1

$(\theta+\Delta\theta)$ = angular distance of bright star from line of sight at time T_2

ι = inclination of system

ε = prolateness of star

ac = semi-axis major of star

a_1 = projected semi-axis major of star on vertical plane at time T_1

a_2 = projected semi-axis major of star on vertical plane at time T_2

then

$$a_1 = ac\sqrt{1-\sin^2\theta\cos^2\iota\varepsilon^2}$$

$$a_2 = ac\sqrt{1-\sin^2(\theta+\Delta\theta)\cos^2\iota\varepsilon^2}$$

If now

$T_2 - T_1 =$ one second of time,

and thus

$\Delta\theta =$ angular velocity of bright star in one second of time,

then, neglecting higher powers of $\Delta\theta$ than the first, the correction to the mean velocity curve at any time T_1 becomes

$$\frac{ac}{2} \cdot \frac{\sin \Delta\theta \sin 2\theta \cos^2 \iota\varepsilon^2}{\sqrt{1 - \sin^2 \theta \cos^2 \iota\varepsilon^2}}$$

The ruling factor in this expression is $\sin 2\theta$. Its existence indicates that the secondary curve crosses the primary at four nodal points.

Its full period is therefore fulfilled in half that of the primary curve.

It may be pointed out that in the remarkable series of researches on the variable star *W Sagittarii* recently carried out by Professor R. H. Curtiss, of the Lick Observatory, the existence of a secondary velocity curve with a period equal to half that of the primary was clearly established.

But before this investigation of Professor Curtiss was undertaken Professor Campbell, in 1901, proved that a similar secondary velocity curve existed in the case of the variable star ζ *Geminorum* (*Astrophysical Journal*, vol. xiii. p. 90).

We give in fig. 2 a representation of the velocity curve of ζ *Geminorum* obtained by Professor Campbell. The evidence of a secondary curve supervening on the primary is indisputable; whether its period is completed in half that of the primary is not so clear.

Although the simple fact that the figure of the revolving components of *W Sagittarii* and ζ *Geminorum* departs considerably from the spherical form will not, and cannot, explain all the secondary velocity variation observed in these stars, the ever-changing figure of the stars must also be taken into consideration; still I am convinced that it will explain the greater part of the bi-period aberration.

I hope in a succeeding paper to enter into a numerical inquiry as regards the satisfactoriness or the reverse of the foregoing partial explanation of secondary radial velocity variation.

I offer it now with the hope that it may give a direction to the thoughts of those who, like the writer, are perplexed at the complete lack of common standing ground, other than a similarity of period, between orbital movement and light variation of certain short-period variables.

At the risk of appearing to wander incontinently away from the subject I have set before me, I might urge that greater confidence would be begotten in the comparison between the velocity

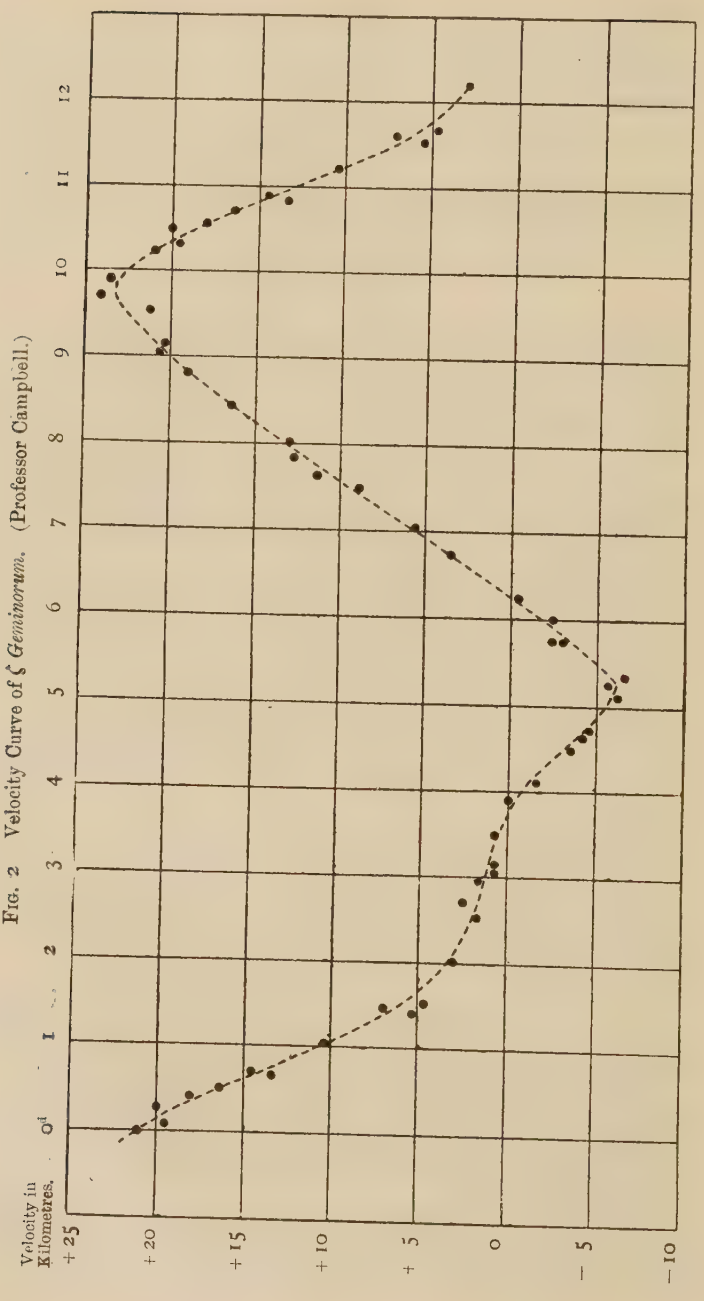


FIG. 2 Velocity Curve of ζ *Geminorum*. (Professor Campbell.)

curve and the light curve of any short-period variable if both curves were obtained synchronously.

To secure a certain velocity curve of a variable star and then, from its elements of variation, to bring up the light curve over possibly a lapse of years is exceedingly unsafe. Professor Curtiss's important correction in the case of *W Sagittarii* should stand as a warning against this mode of treating so important a problem.

What the writer would urge is that velocity observations and light observations should be made, *over the same period of time*, by reputed observers. One could then compare with confidence the two curves, and form a judgment of some weight on the comparison. There would be no necessity in such a case to consider the secular variation of period—always an uncertain factor.

In no way do I desire to detract from, or to throw discredit upon, the comparisons of the conditions of orbital movement and the circumstances of light fluctuations already established. My wish is to obtain as accurate and as absolute a comparison as care, experience, and skill can furnish.

Lovedale: 1906 *February.*

On the Variable Star (38'1905) *RX Andromedæ.*

By A. Stanley Williams.

	h	m	s		°	′	
R.A. =	0	56	26,	Decl. =	+40	31·6	(1855)
„ =	0	58	55	„ =	+40	46·2	(1900)

The few observations of this star obtained here during the latter portion of 1904 and the early part of 1905 seemed to show that it was a variable of the ordinary, regular, short-period (or δ *Cephei*) type, though with an unusually sharply accentuated maximum and a period as long as 45 days. The photographic observations published in the *Astronomische Nachrichten*, No. 4005, indicated, however, the existence of irregularities. Between 1905 July 30 and the end of that year additional observations were made on thirty-four nights,* and these show undoubtedly the presence of very remarkable irregularities. So that, having regard to the usually sharply defined maxima and to the long time during which the star generally remains faint, it would appear that the variable should henceforth be regarded as belonging to the type of *U Geminorum*. The observations upon which this conclusion has been founded are given here in full. It should be premised that these observations are of a more than

* Owing to the unfavourable weather there are many prolonged but unavoidable gaps in the series of observations.

usually satisfactory nature, so that I feel very positive as to the reality of the variations.

A diagram showing the positions of the variable and the comparison stars will be found in the *A.N.*, No. 4005. The following table gives the light-scale and rough magnitudes of the comparison stars used. These rough magnitudes are uncertain as regards their absolute values, but they will serve as a guide to the extent of the variations. They were obtained by assuming the star b to be 10·0 magnitude, and the value of a step to be 0·06 magnitude.

TABLE I.

Comparison Star.	Brightness	Assumed Mag.
b	36·4	10·00
c	21·7	10·88
d	20·7	10·94
e	15·0	11·28

Table II. contains the observations. These were all made with a $6\frac{1}{2}$-inch reflector, with a power of 73, except where otherwise mentioned. The state of the sky is indicated in the fourth column, where IV. implies extreme transparency. The interference from moonlight is also indicated in this column. The last column gives the brightness of the variable according to the light-scale of Table I.

TABLE II.

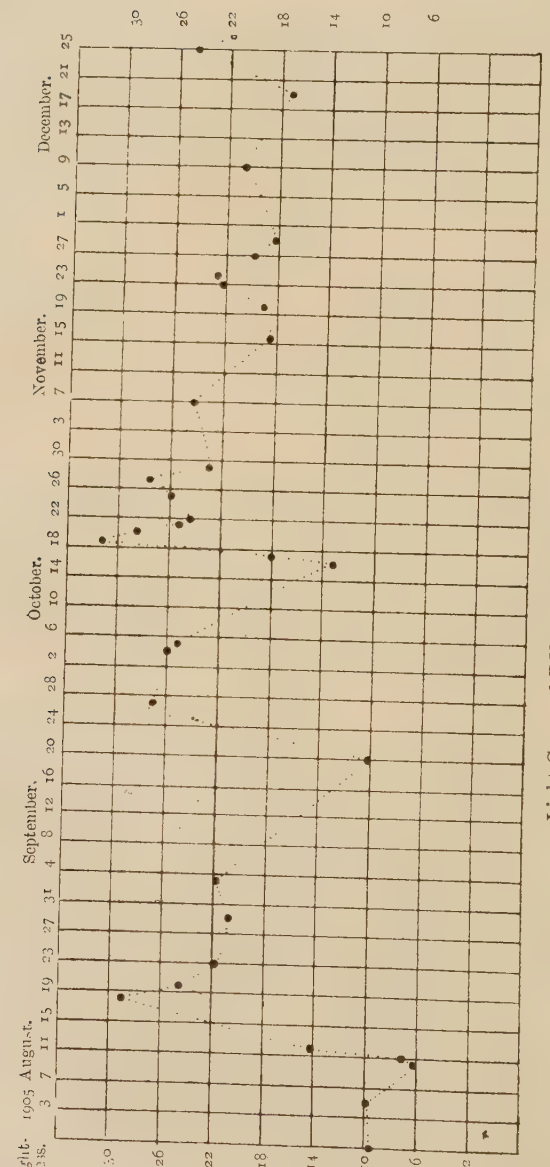
Date. 1905.	G.M.T. h m	Observations.	Sky.	Brightness.
July 30	12 33	c 12 v, d 10 v, e 7 v	IV.	9·5
Aug. 5	14 29	d 10 v, e 6 v	IV.	9·8
10	13 30	d 13 v, e 10 v	IV.	6·3
11	14 44	d 14 v, e 7 v	IV.	7·3
12	14 45	(v 7 d o e)	IV.	14·3
18	15 14	b 10 v 10 c, v 15 d	III. ☽ ☽	29·0
20	14 41	v 3 d, v 12 e, b 15 v 2 c	IV. ☽	24·4
23	14 30	c 2 v 5 d, v 10 e, b 20 v	III. ☽	21·7
29	13 12	d o v 6 e	II.	20·8
Sept. 3	11 10	c o v, v 1 d, v 7 e	III.	21·8
20	15 59	d 11 v, e 4 v	III. ☽ ☽	10·3
27	13 45	b 15 v 7 d, v 12 e, v 7 c	III.	26·8
Oct. 4	9 34	v 5 c, v 5 d, v 12 e, b 13 v	IV.	25·7
4	14 10	v 5 d, v 12 e	IV.	26·3
5	9 26	(c 4 v), b 14 v, v 4 d, v 12 e	IV. ☽	25·3
16	9 48	d 8 v, e 1 v	III. ☽ ☽ ☽	13·3

Date. 1905.	G.M.T. h m	Observations.	Sky.	Brightness.
Oct. 17	11 56	d 4 v 5 e	III. ☽ ☽	18·1
19	10 17	b 10 v 11 d, v 12 c, v 20 e	IV. ☽	31·6
20	10 20	b 15 v 7 c, v 8 d, v 18 e	II.	28·6
21	14 30	b 17 v 4 c, v 4 d, v 12 e	IV. ☽	25·2
22	8 32	b 18 v 2 c, v 4 d, v 12 e	IV.	24·5
25	11 10	b 15 v 7 d, v 15 e, v 7 c	III.	25·8
27	10 14	b 17 v 5 c, v 7 d, v 18 e	III.	27·7
29	10 37	b 18 v 2 c, v 1 d, v 9 e	IV.	23·0
Nov. 7	9 40	v 3 d, v 11 e, b 16 v 3 c	III. ☽ ☽ ☽	24·4
16	9 30	d 4 v 7 e, (d 11 e)	III. ☽ ☽	18·6
20	9 57	d 3 v 8 e	IV.	19·2
23	8 55	d 0 v 12 e, c 0 v, d 12 e	IV.	22·4
24	9 10	v 1 c, v 0 d, v 12 e	III.	22·8
27	7 15	d 2 v 7 e, c 1 v	IV.	19·8
29	7 25	d 4 v 10 e, c 4 v	IV.	18·4
Dec. 9	7 06	v 0 d	III. ☽ ☽ ☽	20·7
19	9 38	d 5 v 3 e	III.	17·1
25	10 16	b 12 v 4 d, v 3 c	II.	24·6
27	9 48	b 18 v, c 1 v 0 d, v 11 e	III.	21·9

Notes.—Aug. 12. Probably should be d 7 v 0 e and so treated; Aug. 29. Some thin cloud about; Sept. 20. Power 110; Oct. 5. c 4 v should probably be v 4 c; Oct. 16. Power 110, sky very bright; Nov. 16. "e very faint to-night?" * Dec. 9. Sky very bright.

The irregular nature of the variations will be seen clearly from the annexed diagram. From minimum brightness on August 11 the star rose rapidly to a sharply defined maximum three or four days later. It was again near minimum brightness on September 20, and a few days later it had risen to a maximum apparently not so sharply defined as the previous one. So far there is nothing very abnormal. But then, after declining to a minimum about October 16, the star rose very rapidly to a very sharply defined maximum on October 19, only about 23 days later than the preceding one. Subsequently to October 19, but subject to minor fluctuations, the star seems to have declined pretty steadily in brightness to December 19, shortly after which date another maximum occurred, probably about December 23. This long decline, during which the variable remained moderately bright, is itself remarkable, though, owing to gaps in the series of observations caused by bad weather, it is just possible that there may have been a maximum somewhere in the interval of 65 days. For the same reason, gaps caused by bad weather, there is an uncertainty of a day or two in the dates of

* The comparison star e may be slightly variable.



Light Curve of *R.X Andromedæ* in the year 1905.

the observed maxima. These may, however, be put as under without likelihood of serious error.

Observed Maximum.	Interval. Days.
1905 Aug. 17	
Sept. 26	40
Oct. 19	23
Dec. 23	65

Two maxima derived from my earlier observations are 1904 December 9 and 1905 January 24, the interval between the two being 46 days. These were both sharply accentuated maxima. The following time-intervals between successive maxima of *SS Cygni*, another variable of the same type, obtained by Dr. E. Hartwig in 1904,* will be interesting for comparison with the foregoing figures—namely, 58, 45, 34, 26, and 36 days. Hartwig further states that small brightenings (*Aufhellungen*) also occur when this star is faint, and that the minimum brightness differs at different epochs, all which might equally well be said to apply to *RX Andromedæ*. In fact, as regards the latter, it would seem that not only are the intervals between successive maxima highly irregular, but that also the star is fainter at some minima than it is at others, and that throughout the course of its changes it is subject to minor variations. And yet there are all through persistent suggestions of periodicity. It is worth noting that the successive minima were each brighter than the one preceding. Thus, August 12, brightness = 6·3; September 20 = 10·3; October 16 = 13·3; and December 19 = 17·1. Two distinct types of maximum were observed by Knott in the case of *U Geminorum*—one a sharply defined one, and the other flatter and more prolonged.† These apparently correspond to the "short" and "long" maxima of *SS Cygni*, as described by Messrs. J. A. Parkhurst and Z. Daniel.‡ The maxima of *RX Andromedæ* of 1905 August 17 and October 19, evidently belong to the former or "short" type, whilst in that of 1905 September 26 we may perhaps recognise the other or "long" type.

Hove: 1906 *February* 28.

* *V. J. S.* 39, Heft 4, p. 260.

† *Journal Liv. Ast. Soc.*, vol. iii. p. 119, and the accompanying diagrams.

‡ *Astrophysical Journal*, vol. xii. p. 267. See also *Popular Astronomy*, vol. vi. p. 159.

Observations of Comet c 1903 from Photographs taken with the 30-inch Reflector of the Thompson Equatorial at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The following positions of Comet *c* 1903 were obtained from photographs taken with the 30-inch reflector. As a rule there were four images on each plate with exposures of two minutes during June, of one minute July 1 to 7, and of thirty seconds to the end. Four reference stars were taken in each case, situated as symmetrically as possible about the comet. The positions of these stars were derived from the catalogues of the Astronomischen Gesellschaft and from *Karlsruhe Observations*, 1885·0.

Date and G.M.T. 1903.				Apparent R.A.			Apparent Dec.			Log Δ.	Correction for Parallax. R.A.	Dec.
d	h	m	s	h	m	s	°	′	″		s	″
June 24	13	16	40	21	49	11·35	− 5	27	58·9	9·8196	− ·33	+ 11·0
26	13	13	34	21	47	58·77	− 3	20	56·6	9·7811	− ·34	+ 11·8
27	12	59	1	21	46	39·41	− 2	10	7·5	9·7619	− ·38	+ 12·1
29	13	14	9	21	43	25·16	+ 0	33	21·4	9·7211	− ·35	+ 12·9
30	13	19	2	21	41	28·67	+ 2	6	26·5	9·6996	− ·34	+ 13·3
July 1	13	9	47	21	39	17·97	+ 3	47	31·9	9·6752	− ·37	+ 13·8
3	12	42	40	21	34	2·59	+ 7	40	10·3	9·6290	− ·46	+ 14·5
6	12	32	24	21	22	55·41	+ 15	11	45·0	9·5582	− ·50	+ 14·8
7	12	26	36	21	18	2·08	+ 18	15	13·8	9·5338	− ·53	+ 14·6
9	12	25	0	21	5	35·73	+ 25	22	45·1	9·5094	− ·50	+ 12·5
10	11	46	54	20	57	51·49	+ 29	21	3·4	9·4885	− ·68	+ 11·8
12	11	44	12	20	36	51·14	+ 38	28	23·0	9·4531	− ·66	+ 7·9
13	11	17	44	20	23	0·73	+ 43	19	4·2	9·4401	− ·79	+ 5·8
15	11	11	4	19	43	37·80	+ 53	17	30·2	9·4268	− ·61	− 0·5
20	11	58	38	16	11	14·71	+ 68	39	9·2	9·4718	+ 2·76	− 1·7
24	9	37	39	13	30	23·01	+ 64	39	6·7	9·5550	+ 2·13	+ 1·9
26	11	34	21	12	45	47·75	+ 60	48	31·3	9·5997	+ 1·82	+ 11·6
30	10	24	34	11	58	49·69	+ 54	12	16·4	9·6855	+ 1·26	+ 10·4
Aug. 1	9	35	39	11	44	26·04	+ 51	27	38·0	9·7254	+ 1·10	+ 9·0
4	9	3	14	11	28	4·58	+ 47	53	19·8	9·7810	+ 0·90	+ 8·3
5	9	19	58	11	23	28·60	+ 46	48	17·8	9·7981	+ ·83	+ 8·8
6	9	26	51	11	19	14·15	+ 45	46	48·3	9·8152	+ ·77	+ 9·0
7	9	12	54	11	15	19·08	+ 44	48	51·2	9·8313	+ ·74	+ 8·6
10	9	30	11	11	4	32·47	+ 42	4	32·1	9·8777	+ ·60	+ 8·8
13	8	45	25	10	54	54·26	+ 39	32	30·9	9·9200	+ ·54	+ 7·7
14	8	48	34	10	51	45·97	+ 38	41	45·1	9·9335	+ ·51	+ ·7

The value of log Δ, used in computing the correction for parallax, was taken from the ephemeris given in *Ast. Nach.* 3881 to July 7, and that in *Ast. Nach.* 3883 from July 9.

Further details of the observations will be given in the Greenwich volume.

1906 *March* 7.

Observations of Comet e 1904 *from Photographs taken with the 30-inch Reflector of the Thompson Equatorial at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

The following positions of Comet *e* 1904 were obtained from photographs taken with the 30-inch reflector. On January 10, 19 and February 11 there was one exposure of twenty minutes on each plate. On January 25, 27 and February 2 there were two exposures of twenty minutes on each plate. Four reference stars were taken in each case situated as symmetrically as possible about the comet. The positions of these were derived from the catalogues of the Astronomischen Gesellschaft.

Date and G.M.T. 1905.				Apparent R.A.			Apparent Dec.			Log Δ.	Correction for Parallax.	
	d	h	m	s	h	m	s	°	′	″		R.A. s	Dec. ″
Jan.	10	6	5	36	1	30	20·45	− 0	21	24·7	0·0837	−0·01	+5·70
	10	6	43	50	1	30	23·00	− 0	20	6·8	0·0837	+0·04	+5·67
	19	6	23	43	1	45	38·11	+ 6	39	5·2	0·1097	+0·04	+4·83
	25	7	3	39	1	57	1·80	+11	6	17·1	0·1267	+0·10	+4·29
	27	6	57	46	2	1	0·26	+12	31	48·8	0·0428	+0·12	+5·05
Feb.	2	6	32	52	2	13	32·02	+16	38	46·0	0·0603	+0·10	+4·41
	11	7	8	2	2	34	10·72	+22	22	20·6	0·0878	+0·16	+3·70

The value of log Δ used on January 10 is taken from the ephemeris given in *Ast. Nach.* 3988, on January 19 and 25 from that in *Ast. Nach.* 3989, and on January 27 and February 2 and 11 from that in *Ast. Nach.* 3990–91.

Further details of the observations will be given in the Greenwich volume.

1906 *March* 7.

Observations of Occultations of Stars by the Moon made at the Royal Observatory, Greenwich, in the year 1905.

(*Communicated by the Astronomer Royal.*)

Day. 1905.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Jan. 10	Disapp. ϕ Aquarii	Thompson Equat. (Hodgson)	100	Dark	5 8 40·14	D. E.
10	,, ,,	Astrographic Equatorial	225	,,	5 8 40·32	W.
10	,, ,,	Sheepshanks Equatorial	100	,,	5 8 40·82	S. D.
10	Reapp. ,,	Thompson Equat. (Hodgson)	100	Bright	6 22 18·71	D. E.
17	Disapp. Bradley 686	Merz Refractor	250	Dark	9 39 27·11	C. D.
17 (*a*)	,, ,,	Astrographic Equatorial	225	,,	9 39 26·52	H. F.
17	,, ,,	Sheepshanks Equatorial	100	,,	9 39 26·73	S. D.
18 (*a*)	,, 130 Tauri	Astrographic Equatorial	225	,,	5 57 50·76	H.
18	,, ,,	Great Equatorial	670	,,	5 57 49·96	H. F.
18 (*b*) (*c*)	,, ,,	Sheepshanks Equatorial	100	,,	5 57 49·88	W.
18	,, ,,	Great Equatorial (Corbett)	120	,,	5 57 49·36	R. C.
18	,, ,,	Old Altazimuth	100	,,	5 57 (52·27)	S. D.
19	,, 26 Geminorum	Merz Refractor	250	,,	4 50 20·06	P. M.
19	,, ,,	Sheepshanks Equatorial	100	,,	4 50 20·84	J. S.
19	,, ,,	Astrographic Equatorial	225	,,	4 50 20·88	W. S.
19	,, ,,	Great Equatorial	670	,,	4 50 21·12	B. E.
Feb. 17	,, W. B (2) VIII. 211	Astrographic Equatorial	225	,,	6 36 19·63	W. S.
17	,, ,,	Sheepshanks Equatorial	100	,,	6 36 21·06	R. C.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer
1905.						
Mar. 12 (*a*)	Disapp. 48 Tauri	Astrographic Equatorial	225	Dark	7 53 36·73	H.
12	,, ,,	Sheepshanks Equatorial	100	,,	7 53 36·80	H. F.
12	,, ,,	Old Altazimuth	100	,,	7 53 (34·50)	J.
16 (*a*)	,, Piazzi VII. 261	Astrographic Equatorial	225	,,	10 4 23·75	H.
16	,, ,,	Sheepshanks Equatorial	100	,,	10 4 24·14	J.
19	,, 56 Leonis	Great Equatorial	670	,,	11 2 9·30	D.
19	,, ,,	Great Equatorial (Corbett)	120	,,	11 2 9·30	B. E.
Apr. 12	,, W. B. (2) VII. 685	Great Equatorial	670	,,	6 38 13·72	W. B.
12	,, ,,	Sheepshanks Equatorial	100	,,	6 38 13·71	W.
12	,, ,,	Merz Refractor	250	,,	6 38 14·02	P. M.
12	,, ,,	Astrographic Equatorial	225	,,	6 38 14·05	W. S.
12	,, ,,	Great Equatorial (Corbett)	120	,,	6 38 13·72	V.
12	Reapp. ,,	Sheepshanks Equatorial	100	Bright	6 45 31·92	W.
12	Disapp. W. B. (2) VII. 761	,, ,,	100	Dark	7 35 34·59	V.
12	,, B. D. + 17°, 1612	,, ,,	100	,,	7 42 49·21	V.
12	,, W. B. (2) VII. 782	Astrographic Equatorial	225	,,	8 0 39·73	W. S.
12	,, ,,	Sheepshanks Equatorial	100	,,	8 0 40·92	V.
12	,, W. B. (2) VII. 796	Astrographic Equatorial	225	,,	8 5 57·86	W. S.
12	,, ,,	Sheepshanks Equatorial	100	,,	8 5 58·37	V.
12	,, W. B. (2) VII. 793	Astrographic Equatorial	225	,,	8 8 54·09	W. S.

Day. 1905.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer
Apr. 12	Disapp. W.B. (2) VII. 793	Sheepshanks Equatorial	100	Dark	8 8 53·98	V.
12	,, B. D. + 17°, 1619	Astrographic Equatorial	225	,,	8 21 18·34	W. S.
12	,, B. D. + 17°, 1616	,, ,,	225	,,	8 23 6·05	W. S.
14	,, W. B. X. 431	,, ,,	225	,,	8 14 7·29	W. S.
14	,, Lalande 18635	,, ,,	225	,,	8 24 16·62	W. S.
15	,, 44 Leonis	,, ,,	225	,,	8 2 43·54	H.
15	,, ,,	Great Equatorial	670	,,	8 2 43·63	H. F.
15	,, ,,	Great Equatorial (Corbett)	120	,,	8 2 43·83	Bies.
15	,, Piazzi X. 67	Astrographic Equatorial	225	,,	8 13 56·88	H.
15 (*d*)	,, ,,	Great Equatorial	670	,,	8 13 56·18	H. F.
15	,, Piazzi X. 67 (Comes)	Astrographic Equatorial	225	,,	8 14 4·06	H.
15 (*b*)	,, ,,	Great Equatorial	670	,,	8 13 59·66	H. F
15	Reapp. 44 Leonis	Astrographic Equatorial	225	Bright	9 13 58·66	H.
17 (*a*)	Disapp. η Virginis	Sheepshanks Equatorial	100	Dark	8 17 11·64	H. F.
17	,, ,,	Astrographic Equatorial	225	,,	8 17 12·35	W. S.
May 9	,, Lalande 14319	,, ,,	225	,,	9 7 3·91	W. S.
10	,, W. B. (2) VIII. 230	Sheepshanks Equatorial	100	,,	9 31 44·91	H. F.
10	,, ,,	Astrographic Equatorial	225	,,	9 31 45·06	W. S.

Day.		Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1905 May	10	Disapp. W. B. (2) VIII. 260	Astrographic Equatorial	225	Dark	10 2 31·91	W. S.
	15 (*d*)	,, 38 Virginis	,, ,,	225	,,	8 29 19·82	H.
	15	,, ,,	Sheepshanks Equatorial	100	,,	8 29 19·95	J. S.
	15 (*c*)	Reapp. ,,	Astrographic Equatorial	225	Bright	8 47 31·72	H.
	15 (*a*)	Disapp. *k* Virginis	,, ,,	225	Dark	11 29 9·36	H.
	15	,, ,,	Sheepshanks Equatorial	100	,,	11 29 9·46	J. S.
Sept.	19	Reapp. θ' Tauri	,, ,,	100	,,	16 36 10·82	J.
	19	,, W. B. (2) IV. 450	,, ,,	100	,,	16 53 43·95	J.
Oct.	4	Disapp. B. F. 2471	Astrographic Equatorial	225	,,	6 36 26·26	W. S.
	4	,, ,,	Sheepshanks Equatorial	100	,,	6 36 26·70	S. D.
Nov.	3	,, Piazzi XX. 250	Astrographic Equatorial	225	,,	5 51 27·08	W. S.
	3	,, O. A. (S) 20742	,, ,,	225	,,	6 7 52·38	W. S.
	3	,, B. D. – 17°, 6081	,, ,,	225	,,	9 10 9·33	W. S.
	6 (*b*)	,, W. B. XXIII. 29	,, ,,	225	,,	5 39 35·82	W. S.
	7	,, 27 Piscium	,, ,,	225	,,	7 29 49·28	H.
	7	,, ,,	Great Equatorial	670	,,	7 29 49·02	H. F.
	7	,, ,,	Great Equatorial (Corbett)	120	,,	7 29 49·22	J. S.
	7 (*a*)	Reapp. ,,	Astrographic Equatorial	225	Bright	8 41 41·58	H.

Day, 1905.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Nov. 7 (*b*)	Disapp. 29 Piscium	Astrographic Equatorial	225	Dark	10 25 59·75	H.
7	,, ,,	Sheepshanks Equatorial	100	,,	10 25 59·92	J. S.
7 (*a*)	Reapp. ,,	Astrographic Equatorial	225	Bright	10 50 39·39	H.
Dec. 9 (*c*)	Disapp, *f* Tauri	,, ,,	225	Dark	4 48 41·42	J. S.
9	,, W. B. III. 569	,, ,,	225	,,	10 13 6·58	W. S.

Notes.

(*a*) Instantaneous. (*b*) Observation doubtful. (*c*) Very faint, foggy. (*d*) Not quite instantaneous.

The apertures of the telescopes used are as follows :—

	inches.		inches.
Great Equatorial	28	Great Equatorial (Corbett Telescope)	$6\frac{1}{2}$
Merz Refractor	$12\frac{3}{4}$	Thompson Equatorial (Hodgson Telescope) ...	6
Astrographic Equatorial (Guiding Telescope) ...	10	Old Altazimuth	4
Sheepshanks Equatorial	$6\frac{3}{4}$		

The initials D., H., C. D., D. E., W. B., H. F., W., J. S., P. M., W. S., R. C., S. D., V., B. E., J., Bies., are those of Mr. Dyson, Mr. Hollis, Mr. Davidson, Mr. Edney, Mr. Bowyer, Mr. Furner, Mr. Witchell, Mr. Storey, Mr. Melotte, Mr. Stevens, Mr. Cullen, Mr. Daniels, Mr. Vagg, Mr. B. Evans, Mr. James, and Mons. Biesbroeck respectively.

Royal Observatory, Greenwich :
1906 *March* 8.

Proposed Plan of the Basic Work of the Perth Observatory.

By W. E. Cooke, M.A.

When we have to design a programme of future work we ought to report our intentions to our "scientific world" and invite criticism. More especially is this the case with a new observatory, and I feel that I owe it to astronomers all over the world to state just what work I definitely propose that the Perth Observatory shall undertake and to invite the fullest criticism.

The Perth Observatory, it must be remembered, is at present only an infant, astronomically speaking. Its environment is not even yet free from disturbing elements, and it has already had several severe attacks, one of which nearly proved fatal. In this latter case its life was probably saved by the kindly intervention of the Royal Society. It will easily be understood therefore, that, with the work of the International Photo-Durchmusterung on hand, my thoughts have hitherto been mainly concentrated upon the immediate present. I have recently, however, had a short breathing space, and think that hopes may not unreasonably be entertained that the necessity for the existence of the Observatory is now recognised by our leading politicians, and that the work will be allowed to proceed undisturbed. Taking, then, for granted that this is an institution whose activities will proceed for centuries, it seems advisable from the start to lay down a programme for a century's work rather than one from year to year. If the Astronomer Royal were asked just wherein the Greenwich observations were so specially valuable, he would doubtless reply that it was because a definite programme (viz. the observation of the positions of the Sun, Moon, planets, and fixed stars) was originally laid down, and has been continued ever since.

We are at present engaged upon the Zone 32°-40° of the International Photo-Durchmusterung, and in connection with that work it has been necessary to select a list of standard stars for observation with the transit-circle. I have endeavoured to find three suitable ones in each square degree; though, of course, this has not been possible in all cases. *This list I propose to observe perpetually*, so that not only will there soon be a catalogue of reference points in this portion of the sky available for immediate use, but, as time goes on, the positions will be determined with greater and greater accuracy, and eventually also the proper motions.

My desire is to work through this list once every ten years, making three determinations of position of each star in each decade. To do this by the ordinary methods of fundamental work would be difficult or impossible with only two computers, who are also the observers, and who have other duties in addition to perform; and it will be necessary to contrive short cuts both for the observing and computing.

1. Professor Auwers has published (in *Astronomische Nachrichten*, Nos. 3431–2 and 4019–20) a "Fundamental-Catalog für Zonen-Beobachtungen am Südhimmel," and the stars of this catalogue situated between 31° and 41° declination will form the basis of reduction both in R.A. and N.P.D.

2. A list of 406 stars, also between those limits, has been prepared to act as secondary standards. These have been selected so as to succeed one another at intervals of three or four minutes in R.A., and so that there shall be at least three in each degree in every two hours of R.A. They include all the fundamental stars.

3. This list, with the addition of a few stars for azimuth, is now being exclusively observed. Clock and instrumental corrections are obtained from the fundamental stars, of which at least six are included in each night's work, and all the positions are thus reduced to the basis of Auwers' "Fundamental-Catalog."

4. It is hoped that one year's observations will give ten determinations of most of these secondary standards.

5. Observations of the main catalogue (of about 10,000 stars) will then be continued, and the instrumental constants will be derived from the positions determined for the secondary standards, of which at least six will be included in each night's work.

6. In this portion of the work the following method is proposed. The ordinary reticule will be replaced by one the idea of which has been suggested by a description of Mr. Hink's plate micrometer. There will be ninety-six horizontal wires in groups of four covering a space of 2°, *i.e.* the lines will be separated by 1′ spaces, but each fifth minute will be missing. The micrometer screw will be cut to about one revolution per minute, so that it need never be moved more than one revolution to pick up a star image in any part of the field. The slipping eyepiece will have a scale for giving the approximate micrometer reading, so that errors of 5′, or multiples of 5′, cannot very well occur. The telescope will be clamped for each evening at some definite degree of declination, and all stars within one degree north or south of this will pass through the field, and can be bisected on one or other of the horizontal wires with only a small turn of the micrometer screw. By this means the necessity for reading the circle will be eliminated, and much time will be saved both in observing and computing. Moreover, one very serious source of error will be eliminated, and the results will probably be more accurate.

7. The R.A. will probably be taken for the present in the usual manner by transit across seven wires, as I cannot afford to purchase a clockwork impersonal micrometer. I have, however, designed one rather different from the usual pattern, which I am hoping to have constructed locally.

8. Owing to the method of observing, the computations will be considerably simplified. The most serious portion of the work is of course the reduction from apparent to mean position.

Several methods, mechanical and otherwise, have already been proposed, and at present honours are about equally divided between Turner's and the Cape tables. I hope soon, in a separate paper, to place before the Society a method differing entirely from either of these.

9. It is hoped that with the present limited staff the whole catalogue (three determinations of each star) may be worked through every ten years. Should this be practicable there will gradually be formed a list of reference points situated between 31° and 41° decl. sufficiently well distributed and sufficiently accurately determined for most purposes for which a catalogue will be required. Moreover, by constantly observing the same stars year by year the observers will feel that not a single particle of their work is being wasted, and that their results, instead of becoming out of date and lost on library shelves, will become increasingly valuable decade after decade.

10. Observations of the zones 31°–34°, 39°–40° have been already made upon older-established principles, using *Nautical Almanac* clock-stars and nadir points ; but these can probably be reduced to the new system. It may happen that an interruption to the whole scheme will shortly occur, as I have not yet been able to start measuring the plates for the Astrographic Durchmusterung. The whole of the catalogue plates will shortly be completed, and I may have to suspend all night-work in order to get them measured. An interruption of a few years, however, will make little difference to such a scheme as I have proposed.

The Observatory, Perth, Western Australia:
1905 *November* 11.

Errata in Annual Report.

Vol. lxvi. p. 176, line 9 from bottom, *for* transit of *Mercury read* transit of *Venus.*

Page 217, line 21 from bottom, *for* Oello *read* Oello.
„ 217, „ 17 from bottom, *for* Genna *read* Genua.
„ 254, diagram of *Polaris, for* 1898 August *read* 1899 August.